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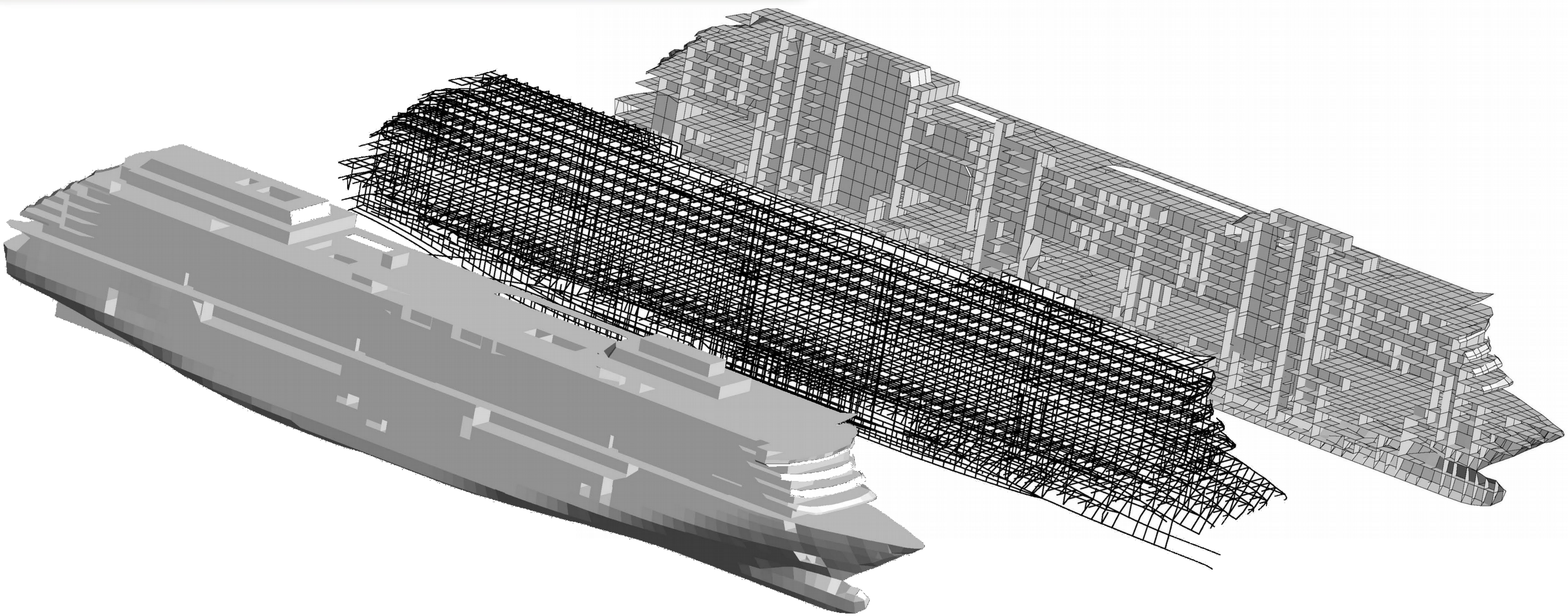
HLME602
Structures et dimensionnement

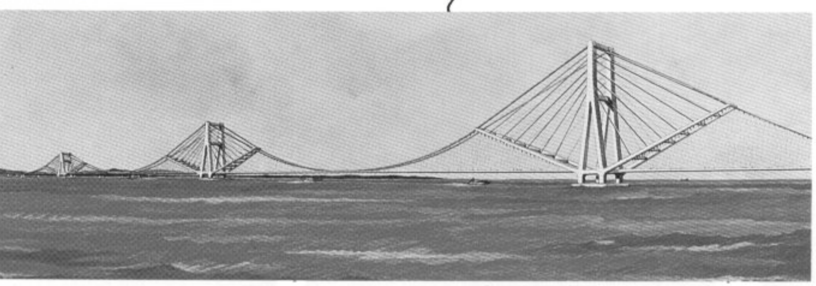
Partie Dimensionnement de Poutres

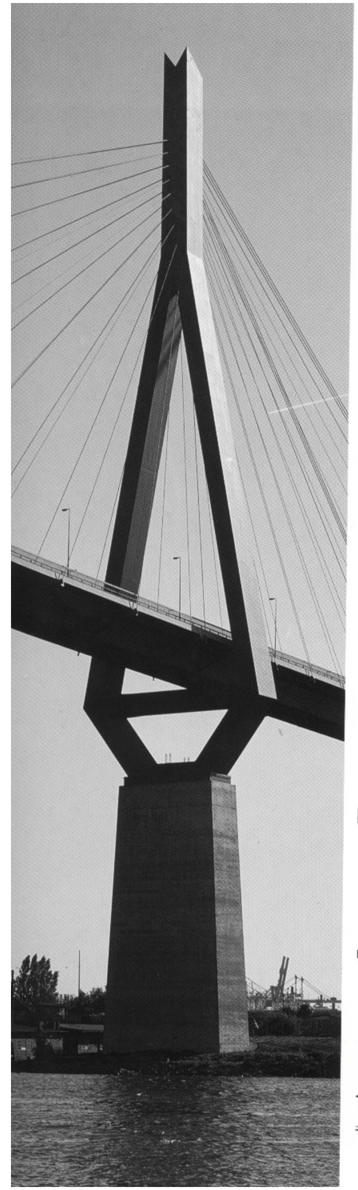
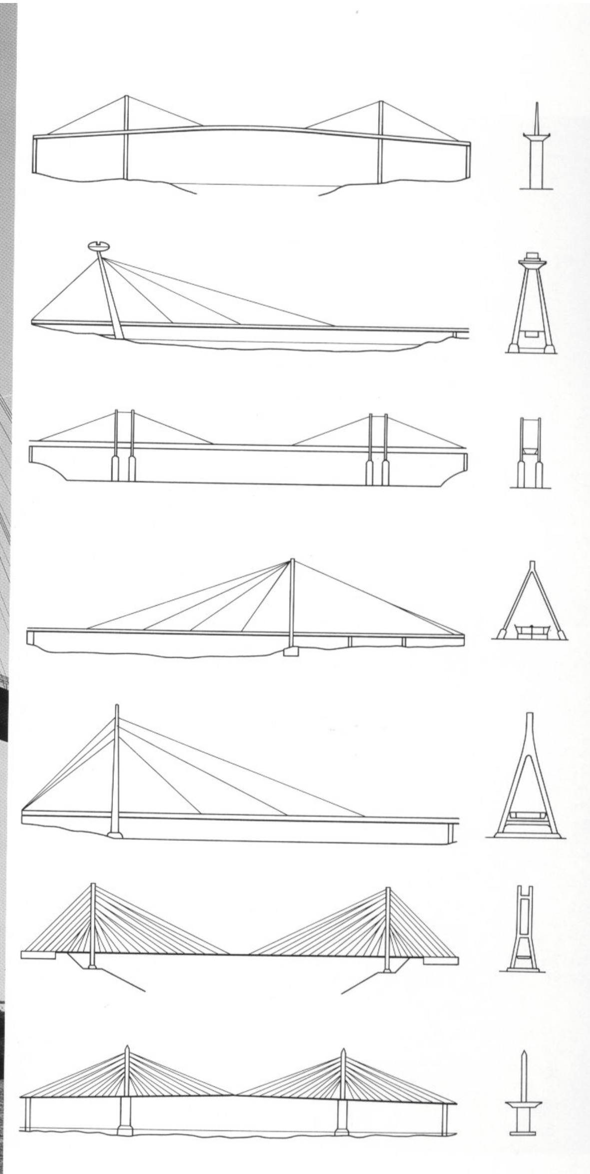
Plan du cours

I. Introduction : rappels de RdM et critères de dimensionnement
II. Déformée des poutres courbes : formules de Bresse
III. Flambement des poutres droites
IV. Dimensionnement à la limite élastique : quelques sollicitations élémentaires


Courtesy of Principia Marine



Le dimensionnement selon le papa de Calvin

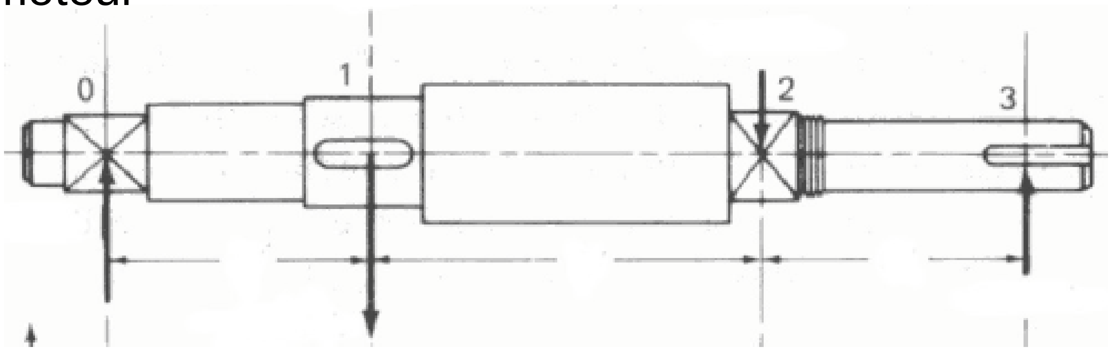
Des essais

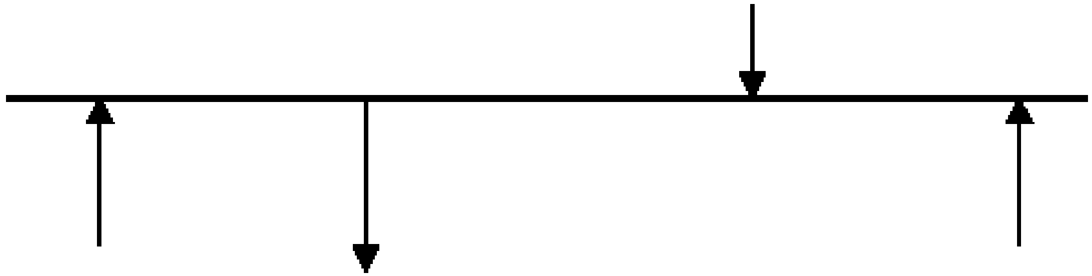
Pont de Normandie

essai de chargement avant mise en service (80 camions = 1300 tonnes)

Un modèle

Arbre moteur





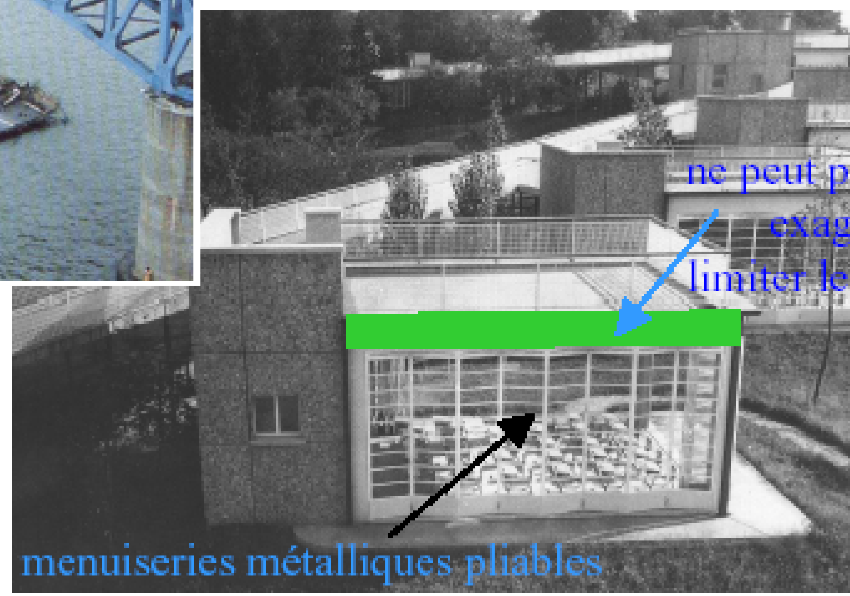
Son modèle en RdM

pas de prise en compte des concentrations de contraintes…

Des critères

Résistance

Rigidité



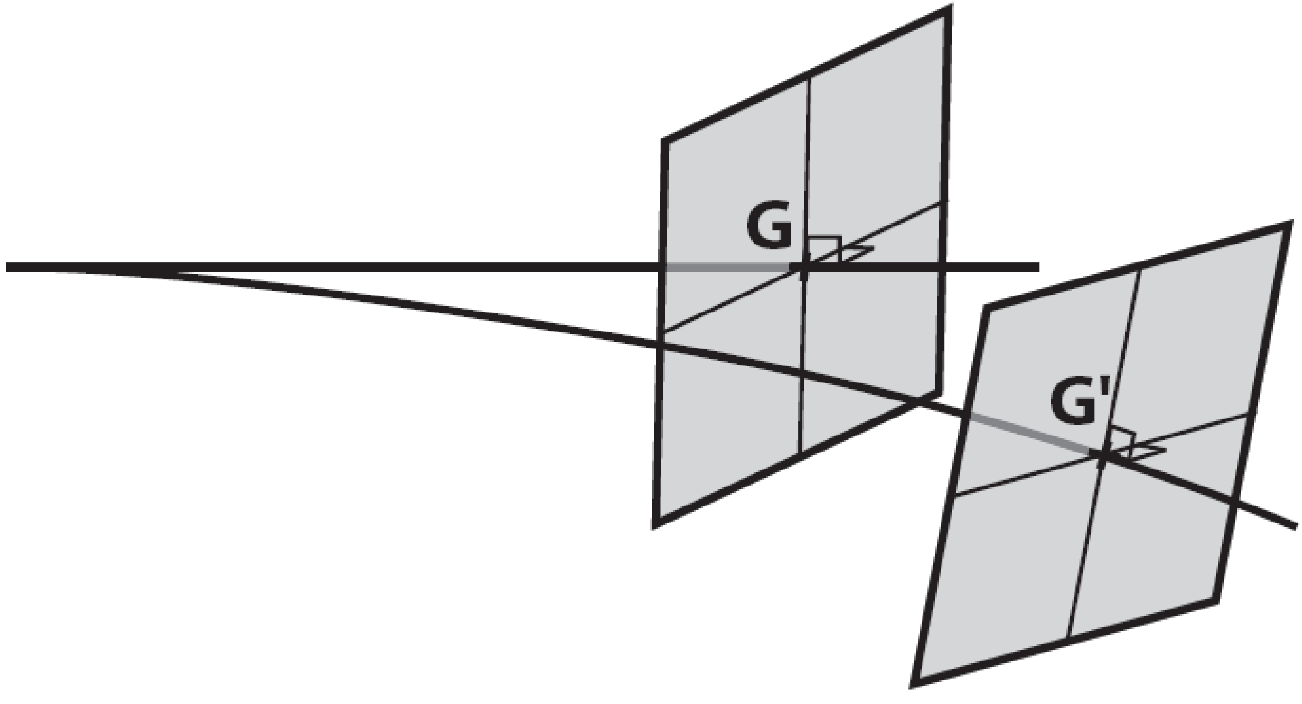
E. Beaudoin & M. Lods, Ecole en pleine air à Suresnes, 1932-35

P. Gössel & G. Leuthaûser, L'architecture au XXème siècle, Taschen, 1991

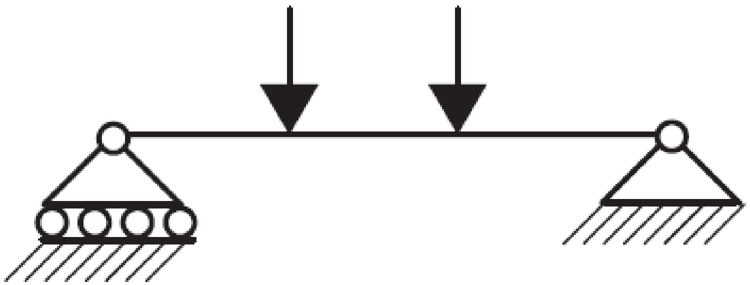
Mais aussi : instabilités (par exemple : **le flambement**)

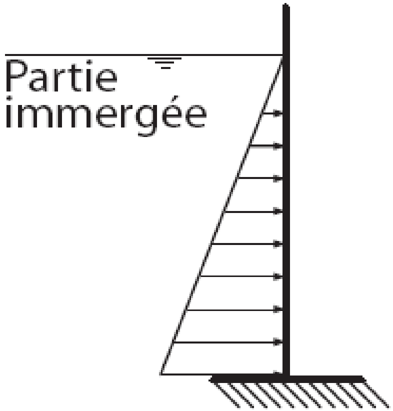
Rappels de RdM

Hypothèse de Navier-Bernoulli



Efforts extérieurs

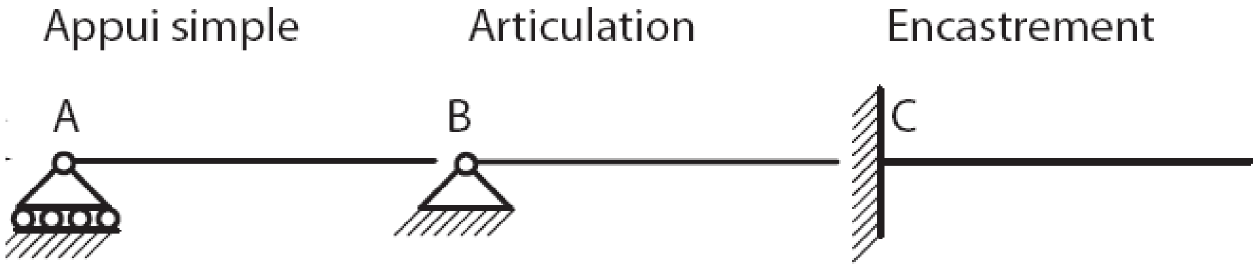




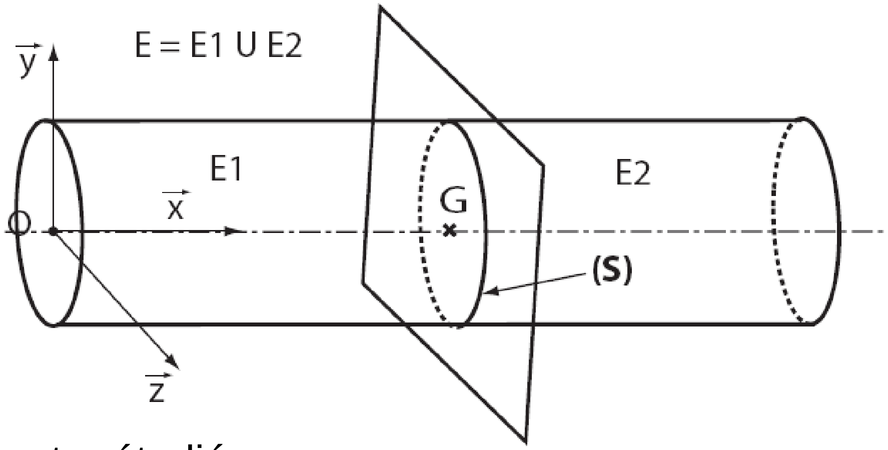
Actions concentrées des roues d'un véhicule sur une poutre

Actions réparties de l'eau sur un barrage

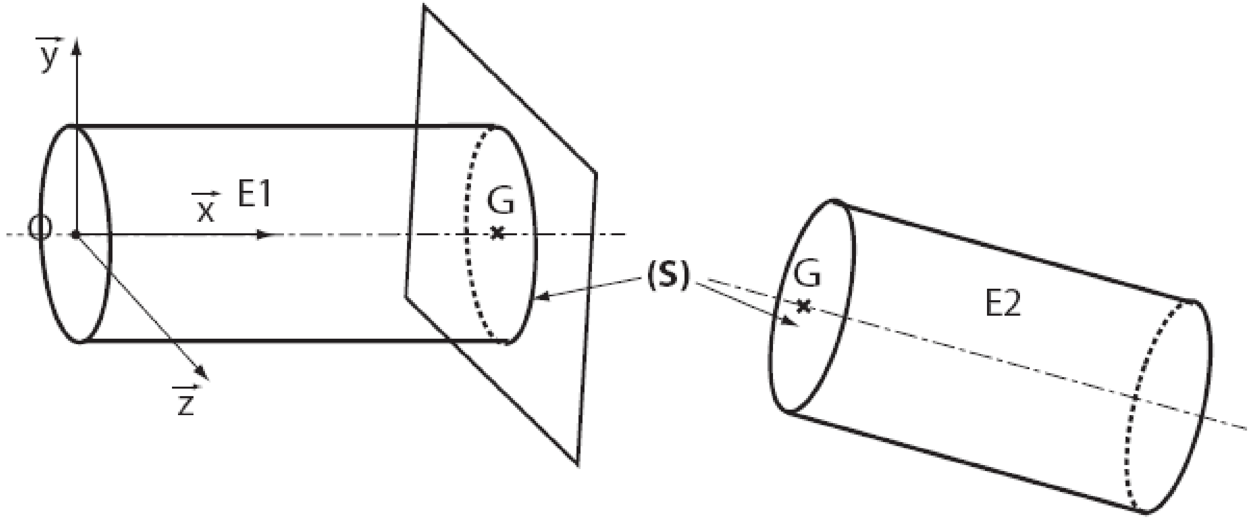
Liaisons



Efforts intérieurs



Poutre étudiée



Poutre séparée en 2 par la pensée

Sollicitations élémentaires

Traction / compression $\quad T = \begin{Bmatrix} N\vec{x}_1 \\ \vec{0} \end{Bmatrix}_G$ $\quad$ $N > 0$ traction, $N < 0$ compression

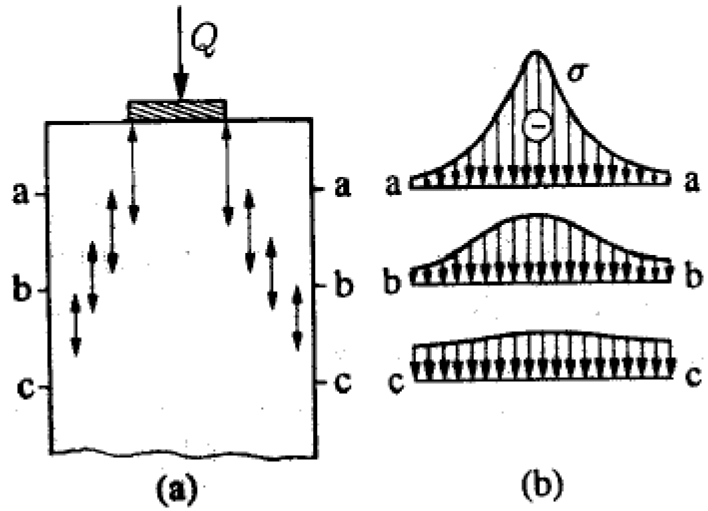
Flexion pure $\quad T = \begin{Bmatrix} \vec{0} \\ M_{f2}\vec{x}_2 + M_{f3}\vec{x}_3 \end{Bmatrix}_G$

Flexion simple $\quad T = \begin{Bmatrix} \vec{T}_3\vec{x}_3 \\ M_{f2}\vec{x}_2 \end{Bmatrix}_G + \begin{Bmatrix} \vec{T}_2\vec{x}_2 \\ M_{f3}\vec{x}_3 \end{Bmatrix}_G$

Cisaillement pur $\quad T = \begin{Bmatrix} T_2\vec{x}_2 + T_3\vec{x}_3 \\ \vec{0} \end{Bmatrix}_G$

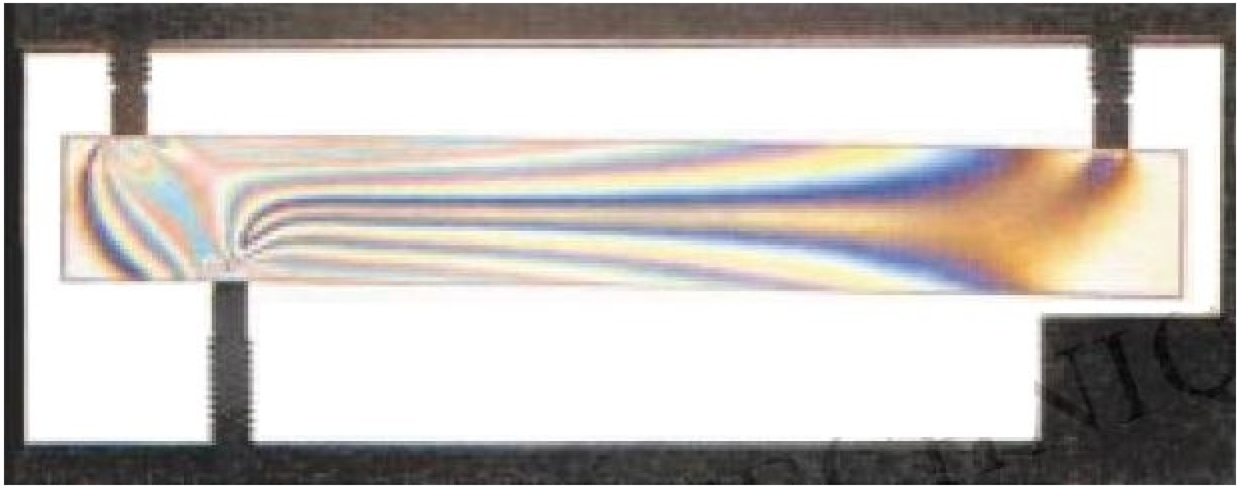
Torsion pure $\quad T = \begin{Bmatrix} \vec{0} \\ M_t\vec{x}_1 \end{Bmatrix}_G$

Principe de Saint-Venant



Principe de Saint-Venant

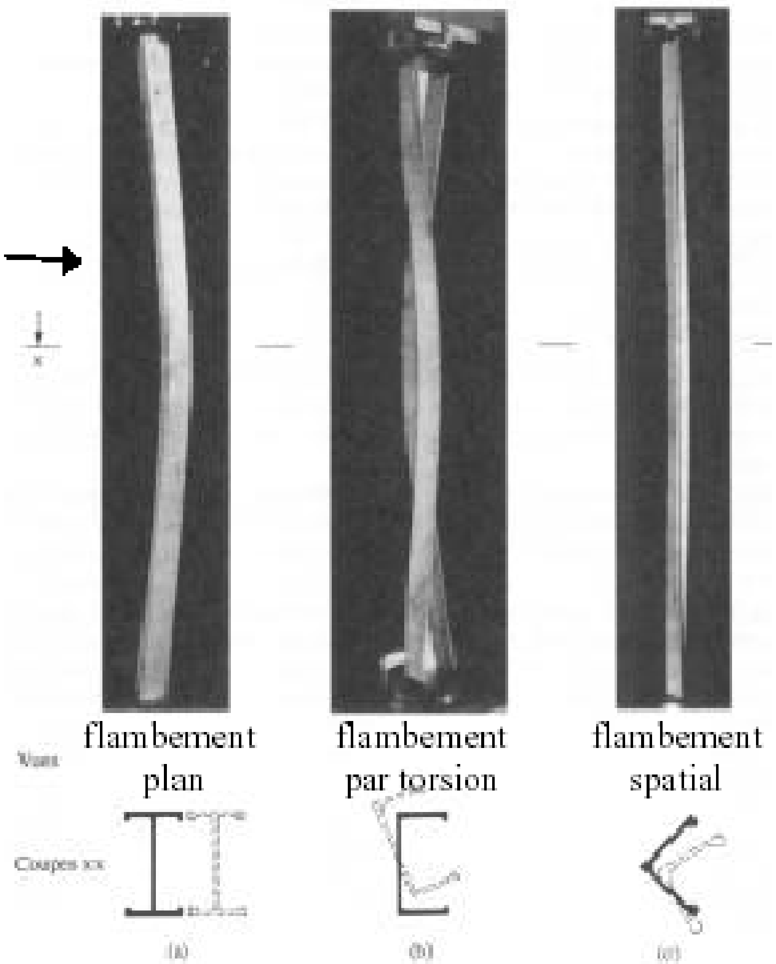
Poutre en flexion simple



source : J. Salençon, École Polytechnique

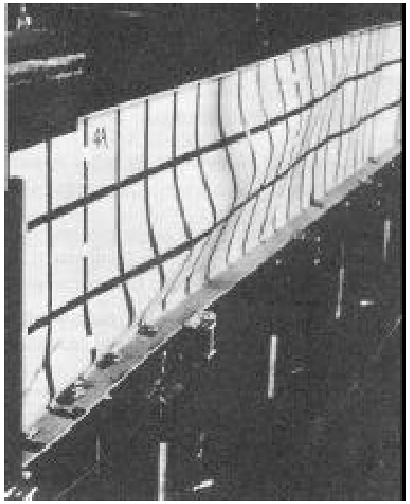
Flambement des poutres droites

seul cas étudié ici



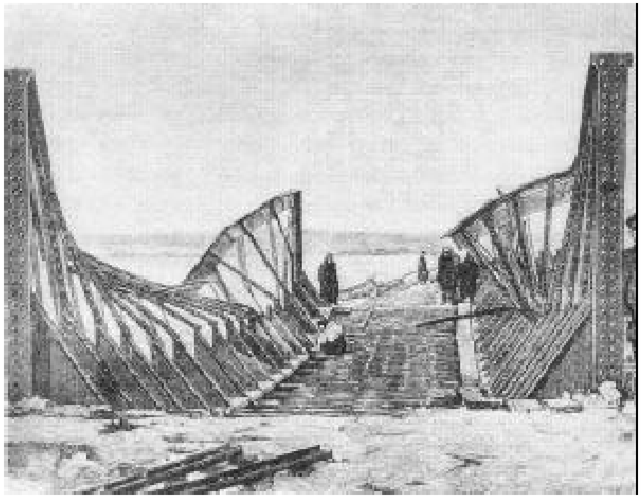
Des instabilités

Exemple d'instabilité locale

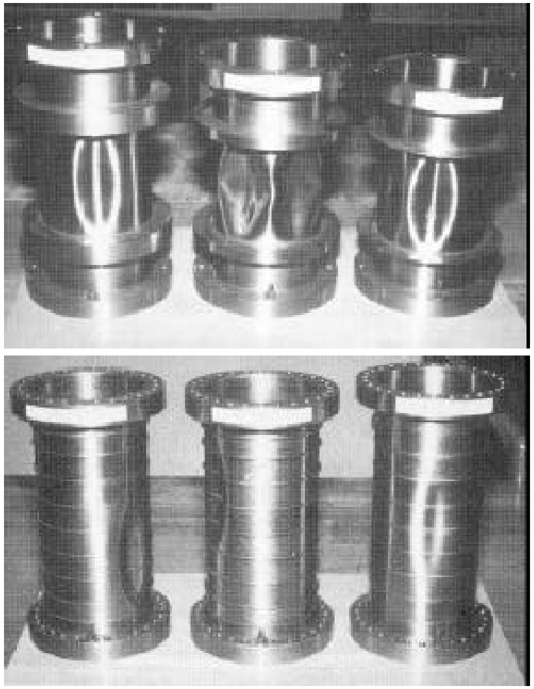


voilement de l’âme mince
d’une poutre en acier
(effet de l’effort tranchant)

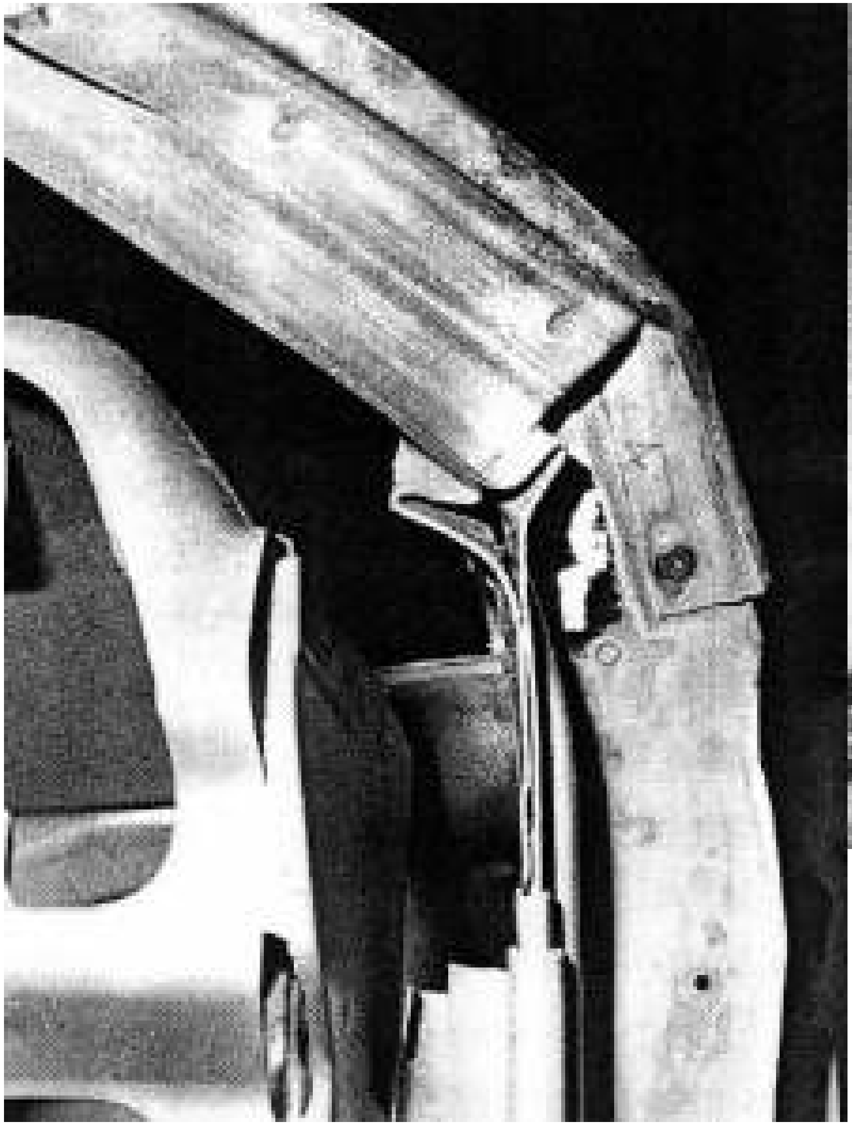
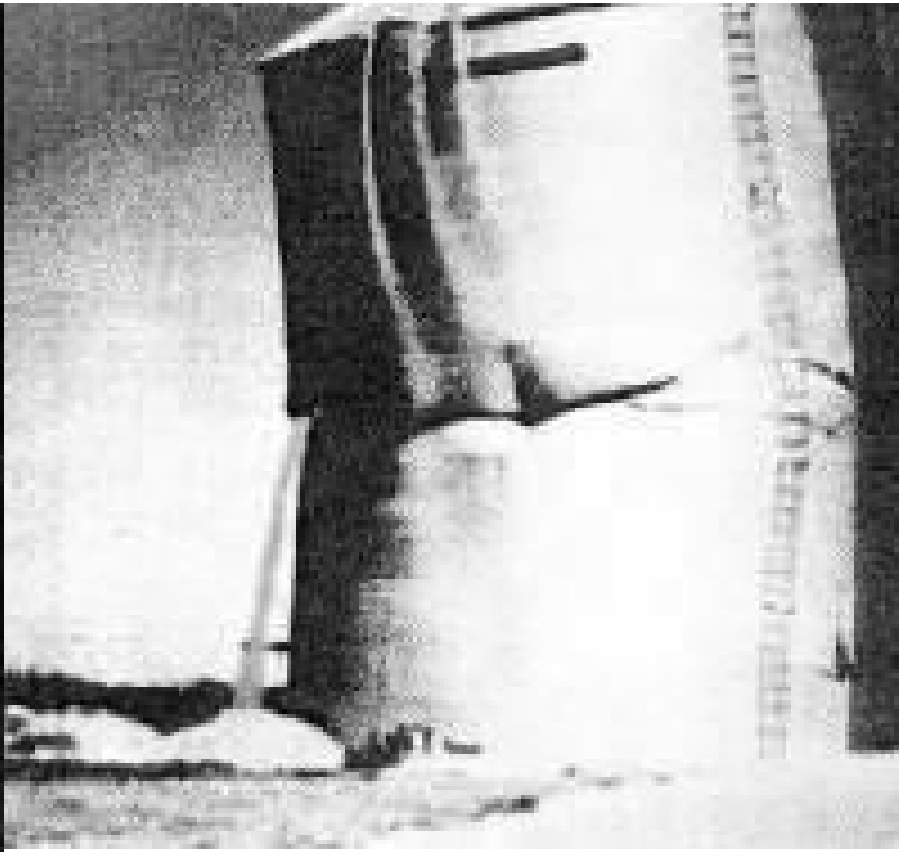
Exemple d'instabilité globale

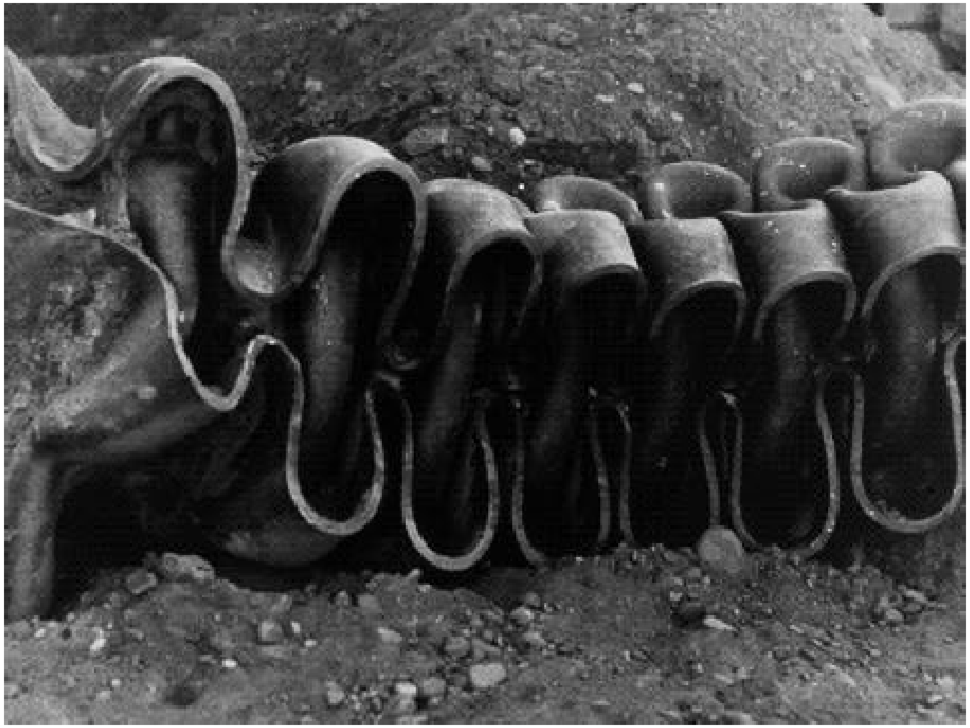


flambement d’ensemble de la
membrure supérieure des
poutres en treillis d’un pont de
chemin de fer
(Russie, vers 1890)



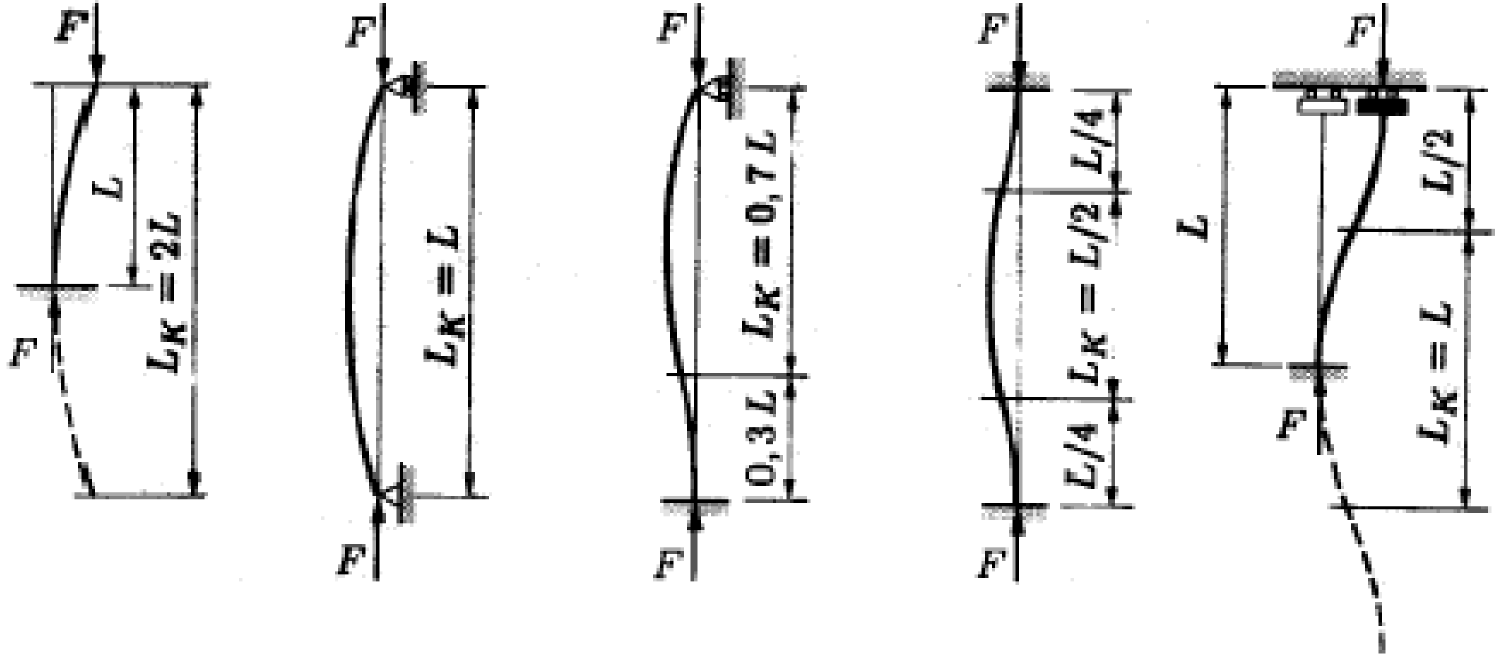
Des instabilités



Longueur de flambement équivalente

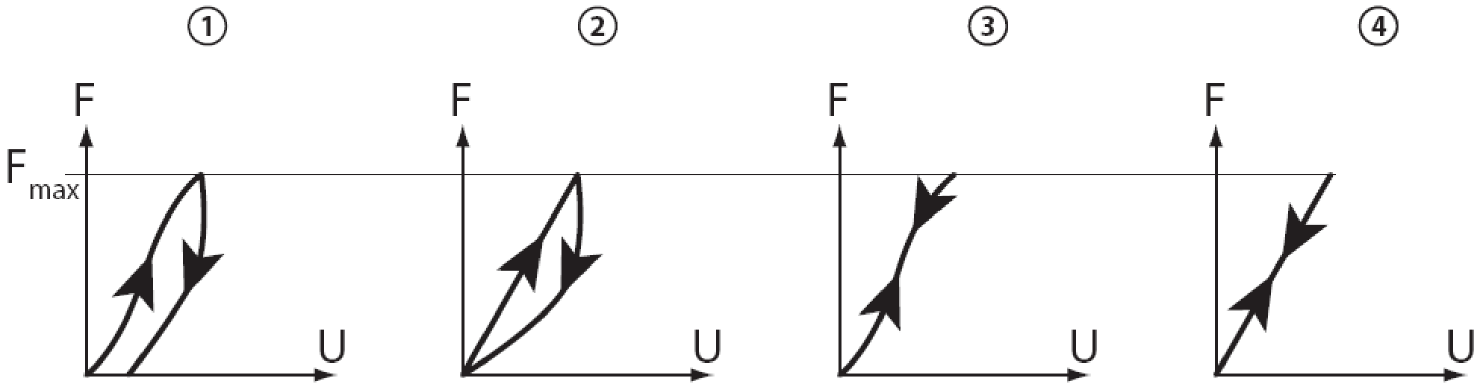
esquisser le premier mode (respect des appuis !)



Dimensionnement à la limite élastique

Traction compression

Élasticité linéaire Courbes effort-déplacement pour différents ressorts



~~élastique~~

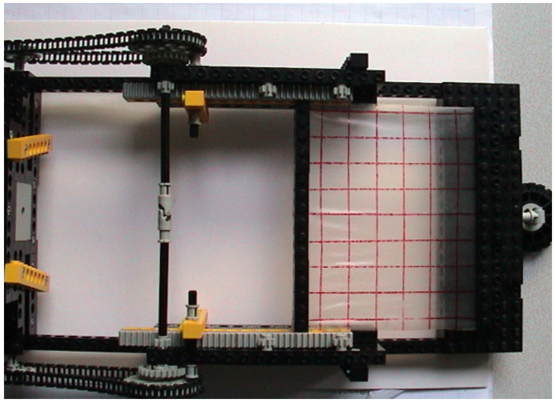
~~élastique~~

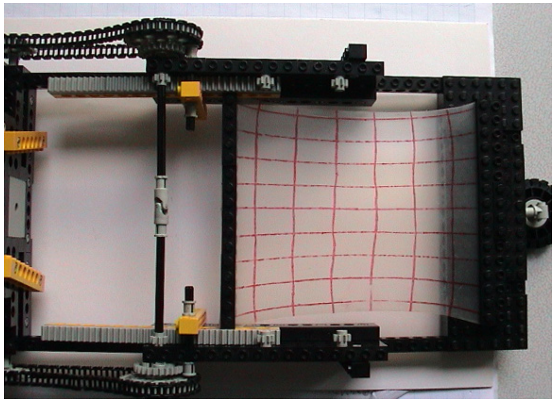
élastique
~~linéaire~~

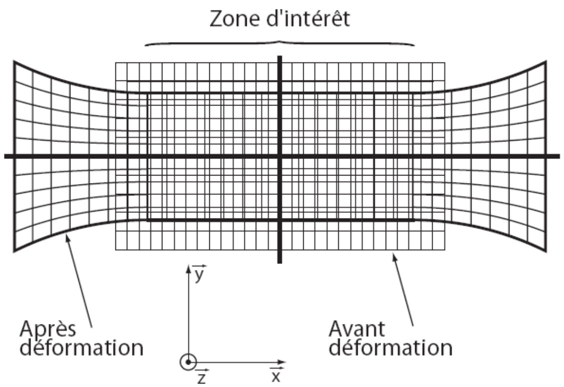
élastique
linéaire

Traction compression

Méthode des grilles

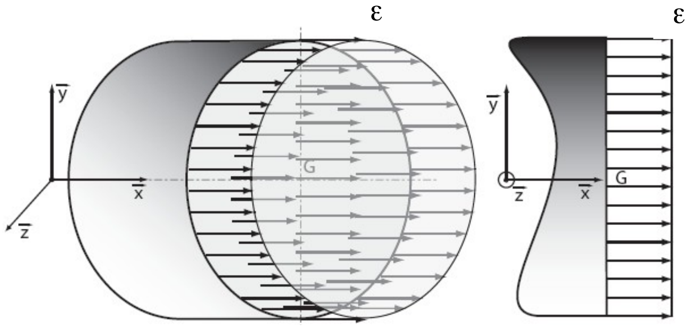


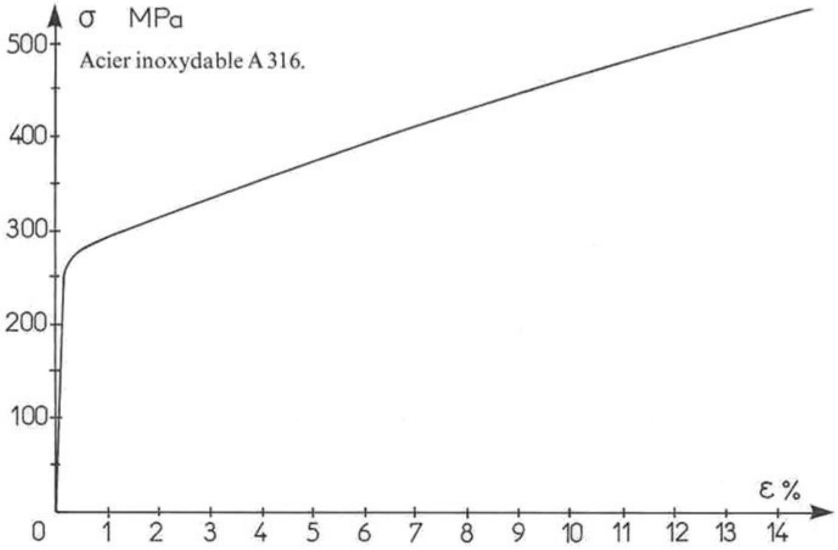




source :P.-A. Boucard, Cachan

Déformations axiales dans la section

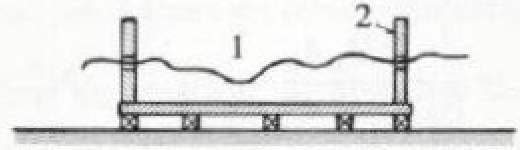
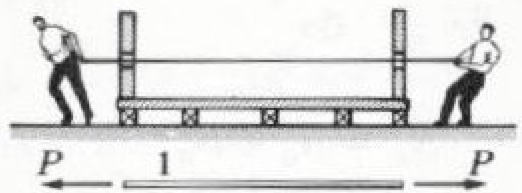
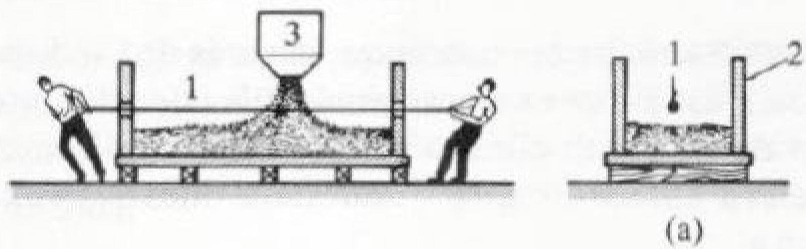
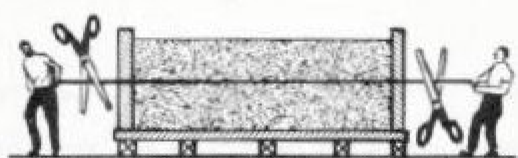




source : Lemaître, Chaboche, *Mécanique des Matériaux Solides*, Dunod, 1988

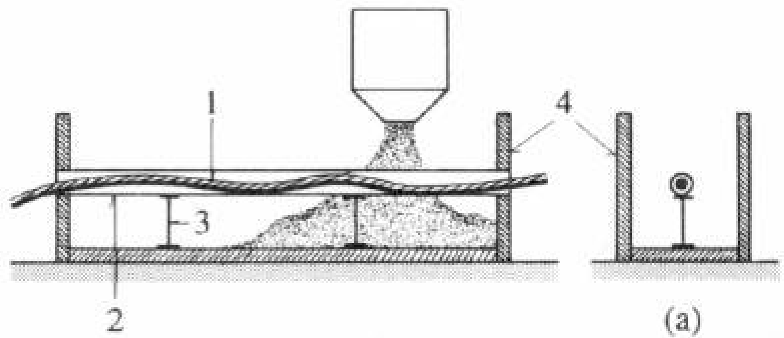
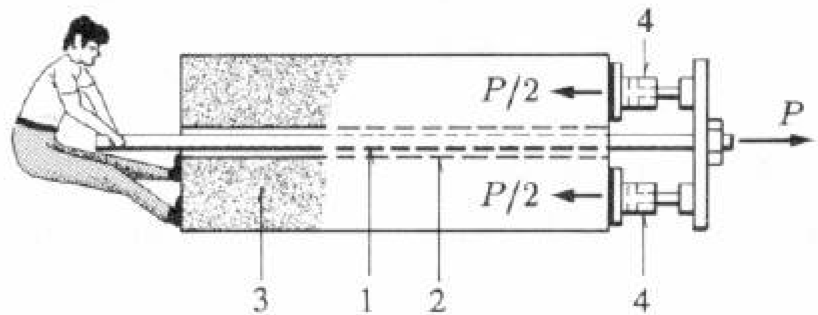
Traction-compression sur bi-matériau

pré-tension dans le béton

Etapes	
1 1 : fil non tendu 2 : coffrage	
2 Mise en tension du fil *en prenant appui sur le sol* 1 : fil précontraint	
3 Bétonnage de la pièce (a) coupe 1 : fil 2 : coffrage 3 : béton	
4 On coupe le fil après durcissement du béton	

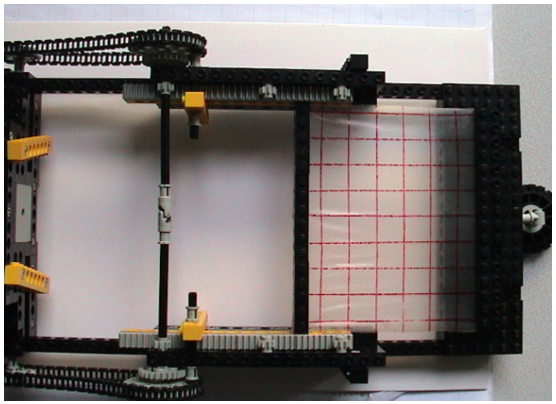
Traction-compression sur bi-matériau

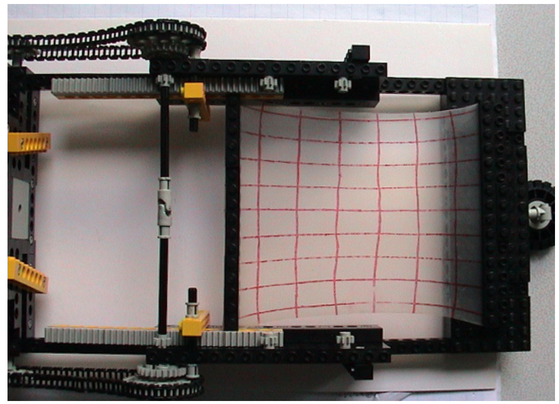
post-tension dans le béton

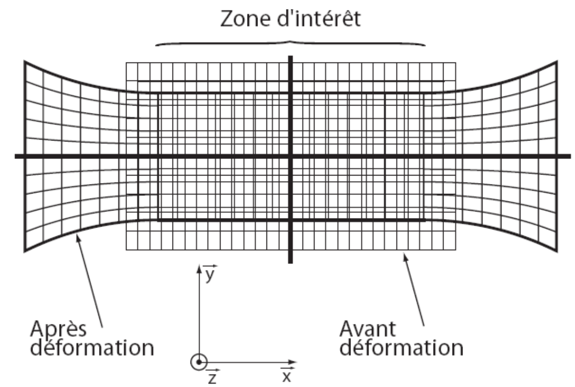
Etapes	
1 Bétonnage de la pièce (a) coupe 1 : câble non tendu 2 : gaine 3 : support de la gaine 4 : coffrage	
2 Mise en tension du câble *en prenant appui sur la section de béton*, après durcissement du béton 1 : câble 2 : gaine 3 : béton 4 : vérin	
3 Blocage du câble aux extrémités (divers systèmes de blocage) et injection	

Traction compression

Méthode des grilles

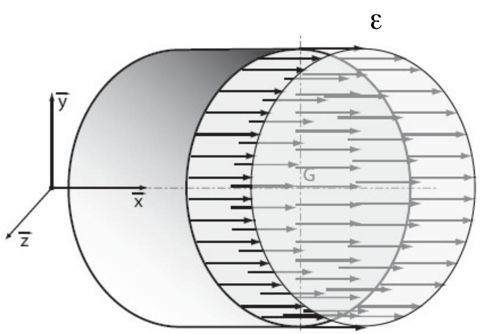


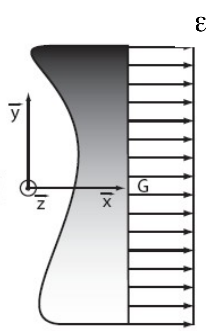


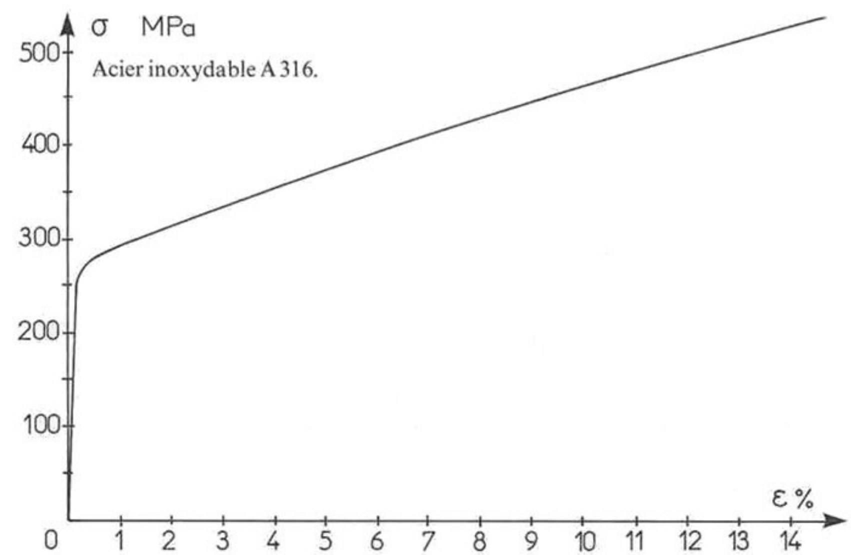


source :P.-A. Boucard, Cachan

Déformations axiales dans la section



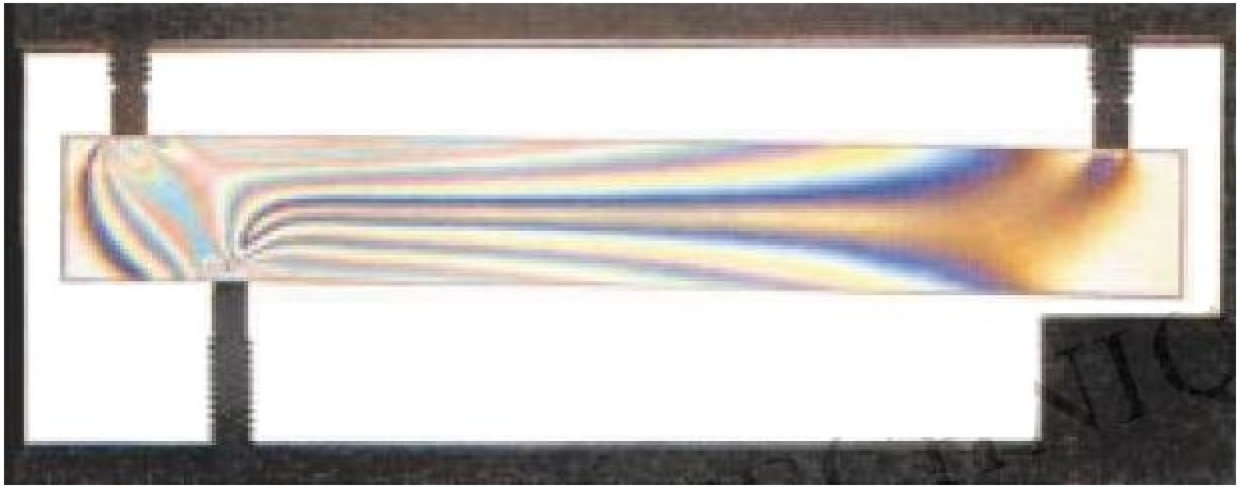




source : Lemaître, Chaboche, *Mécanique des Matériaux Solides*, Dunod, 1988

Principe de Saint-Venant

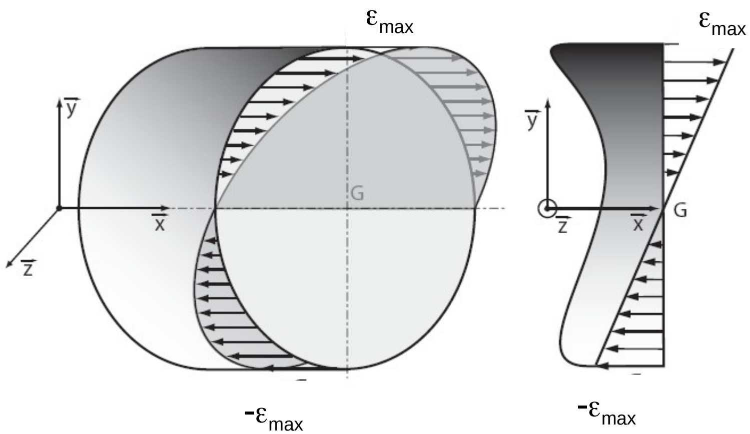
Poutre en flexion simple



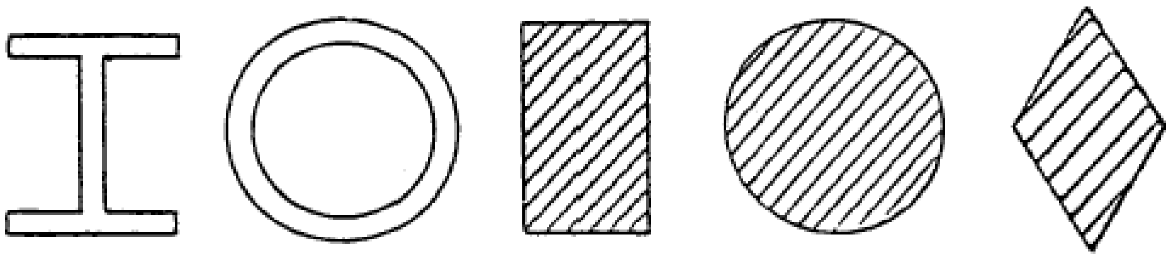
source : J. Salençon, École Polytechnique

Flexion pure

Déformations axiales dans la section



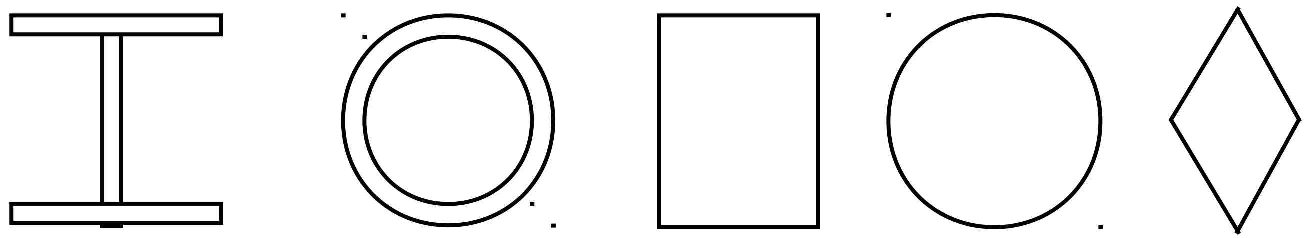
Rendement géométrique en flexion



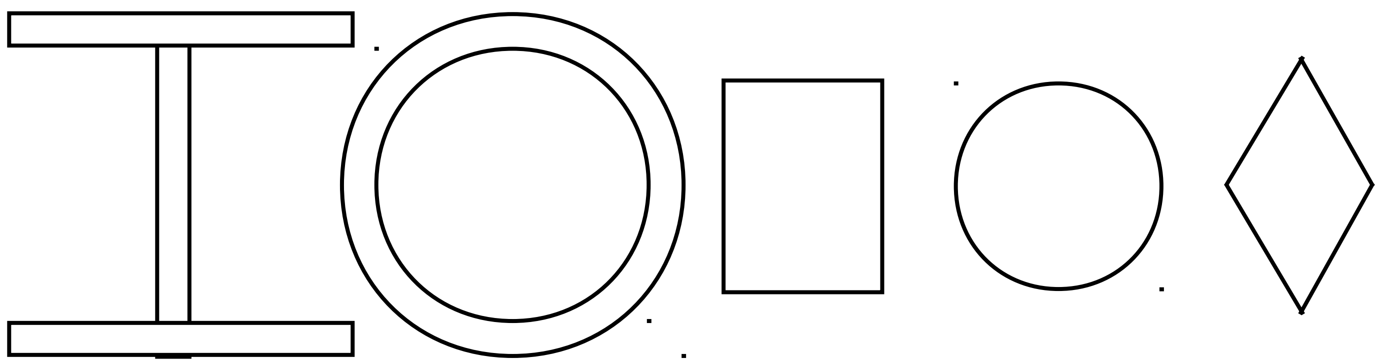
$\eta \approx 2/3$ $\eta \approx 1/2$ $\eta \approx 1/3$ $\eta = 1/4$ $\eta \approx 1/6$

Rendement géométrique en flexion

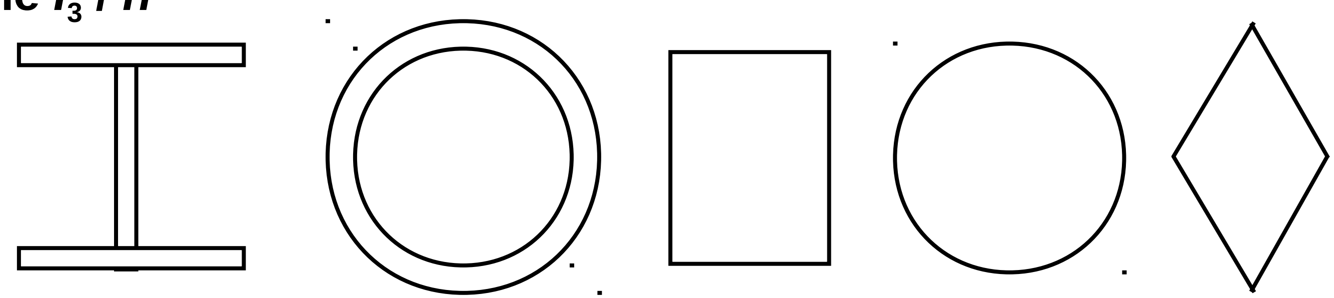
Même hauteur *h*



Même section *S*

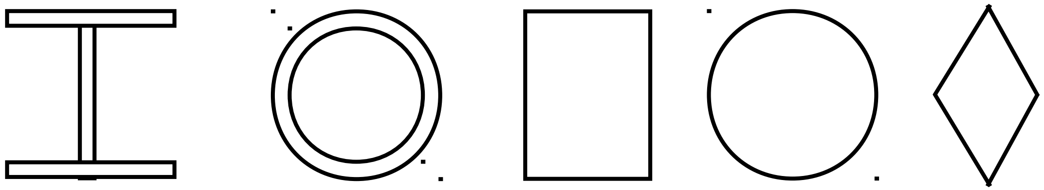


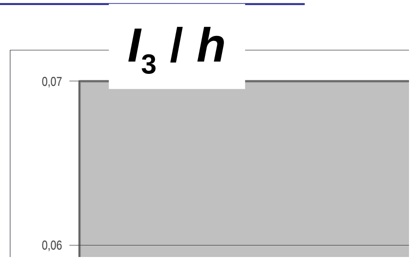
Même I_3 / *h*

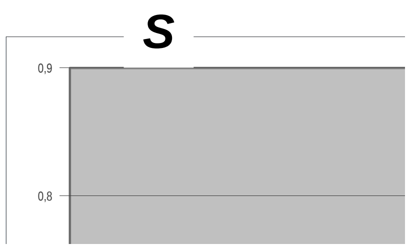


Rendement géométrique en flexion

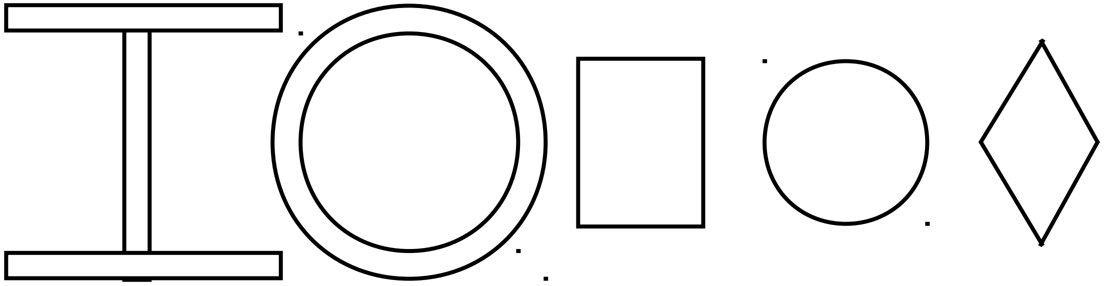
Même hauteur *h*

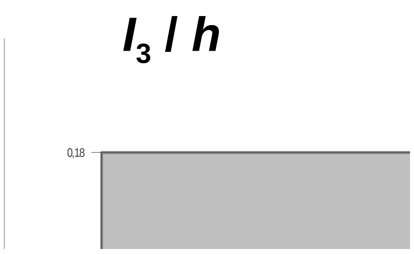




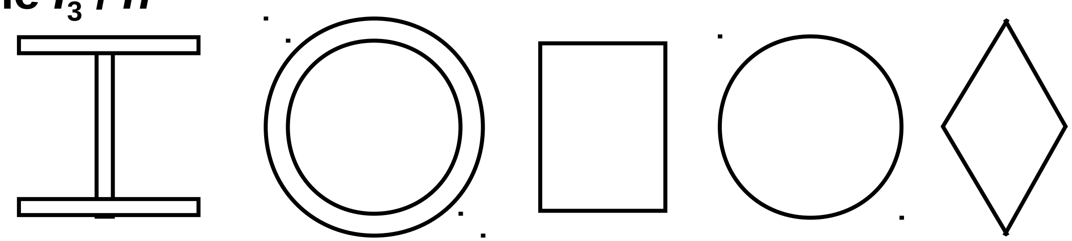


Même section *S*





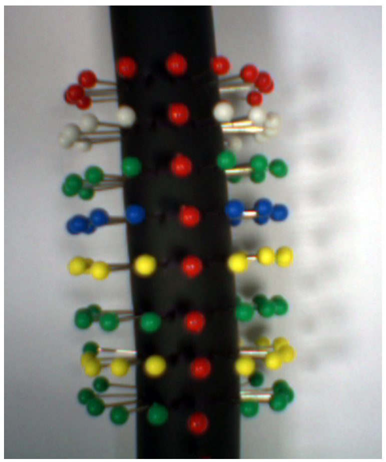
Même I_3 / h

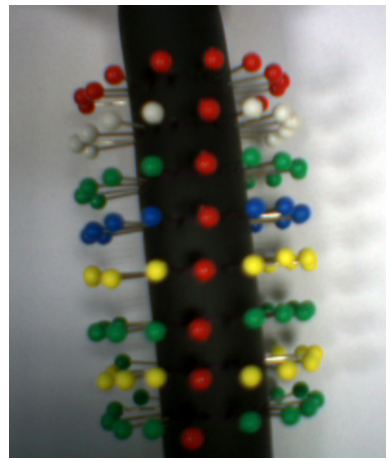


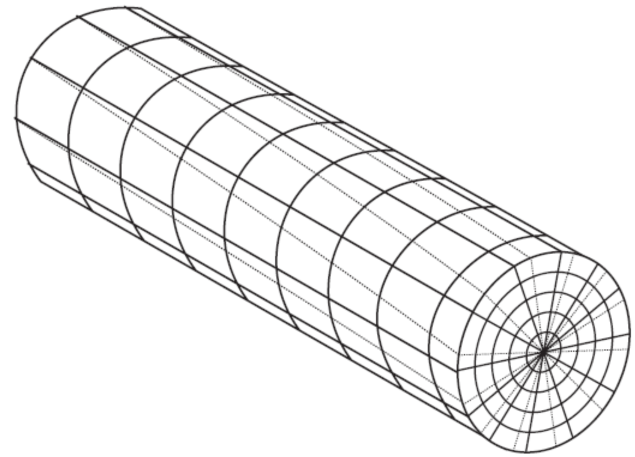
Torsion

Limitation : poutres droites à section circulaire
(sinon, les sections se gauchissent…)

Méthode des “grilles”

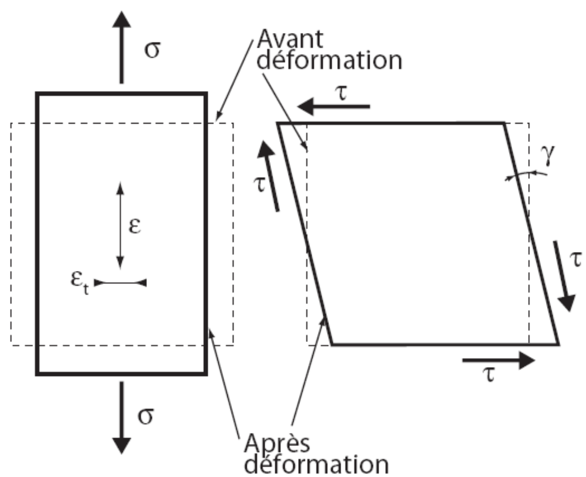


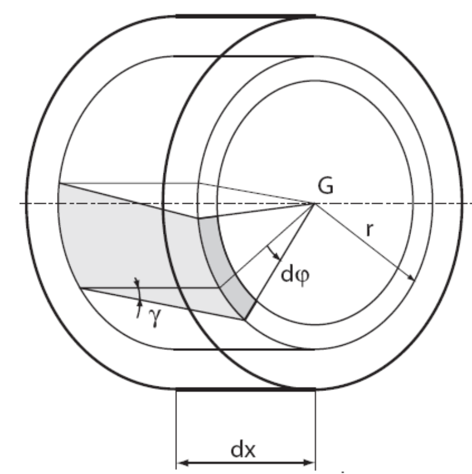




source :P.-A. Boucard, Cachan

Déformations
longitudinale et
transverse
et de cisaillement





Torsion

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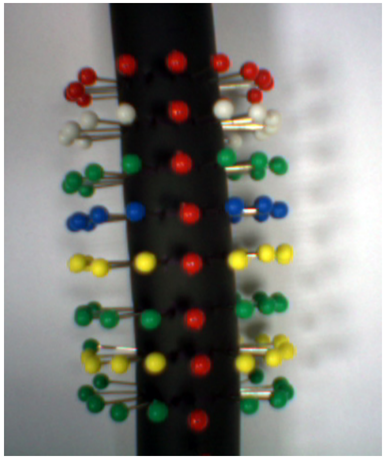
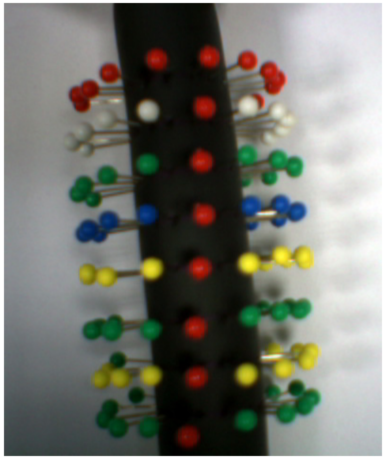
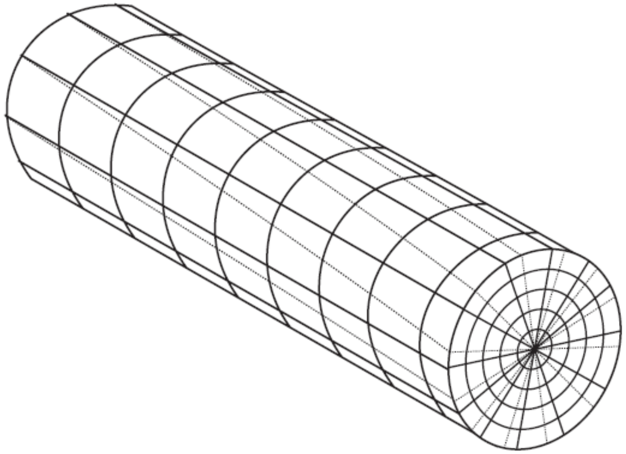
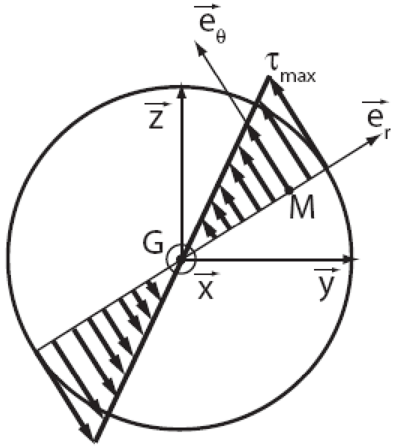


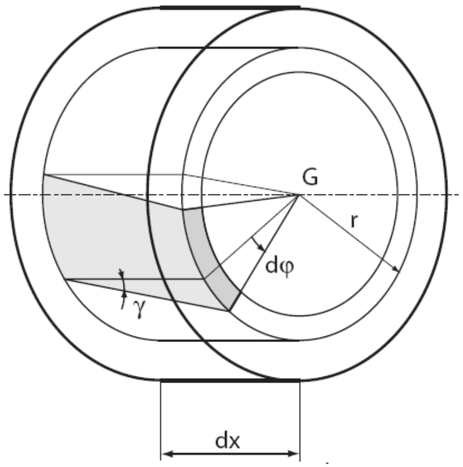




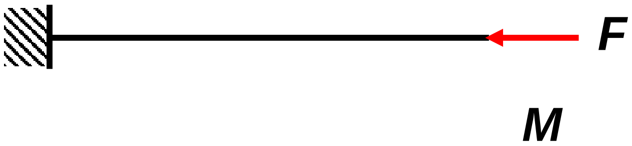
source :P.-A. Boucard, Cachan

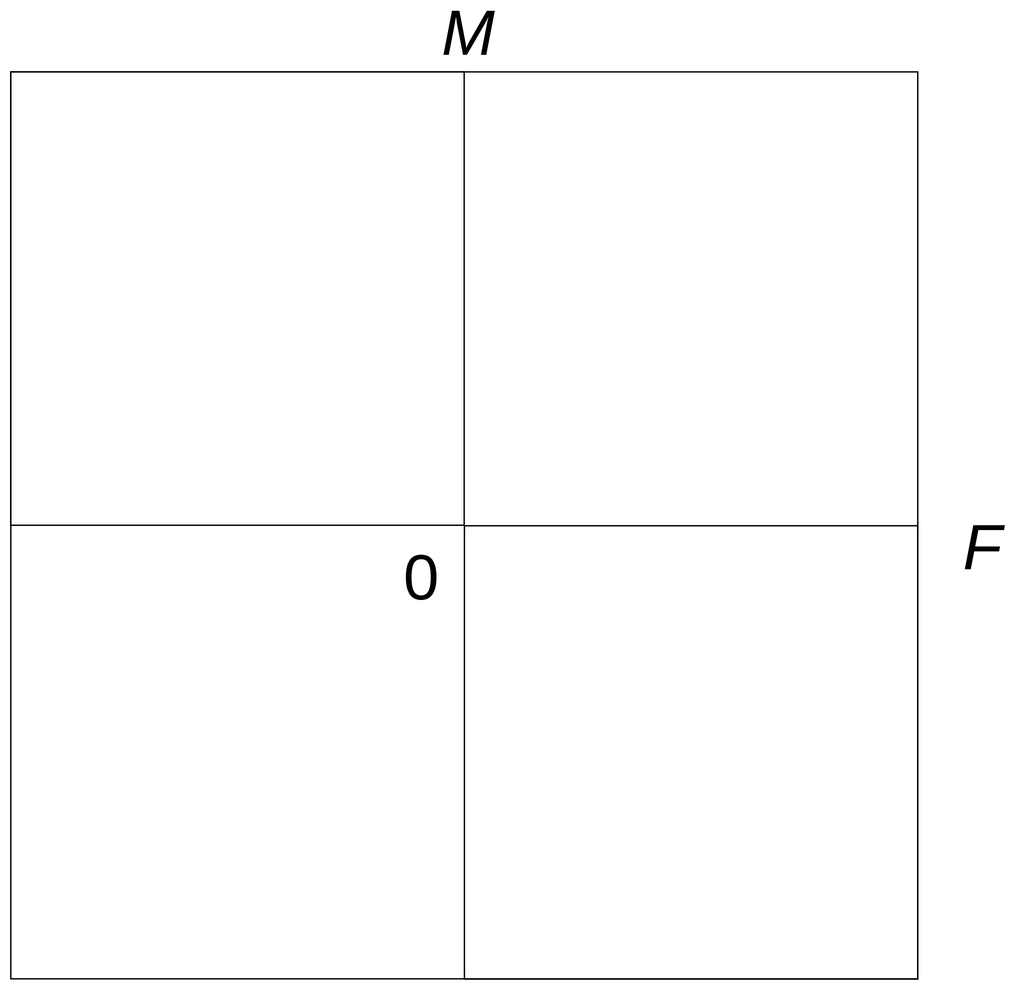
Déformations de cisaillement dans la section





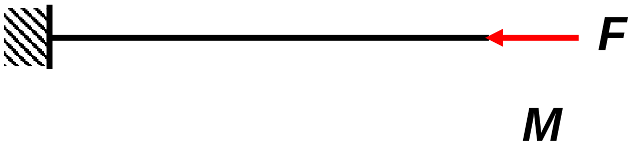
Exemple : domaine limite – poutre console

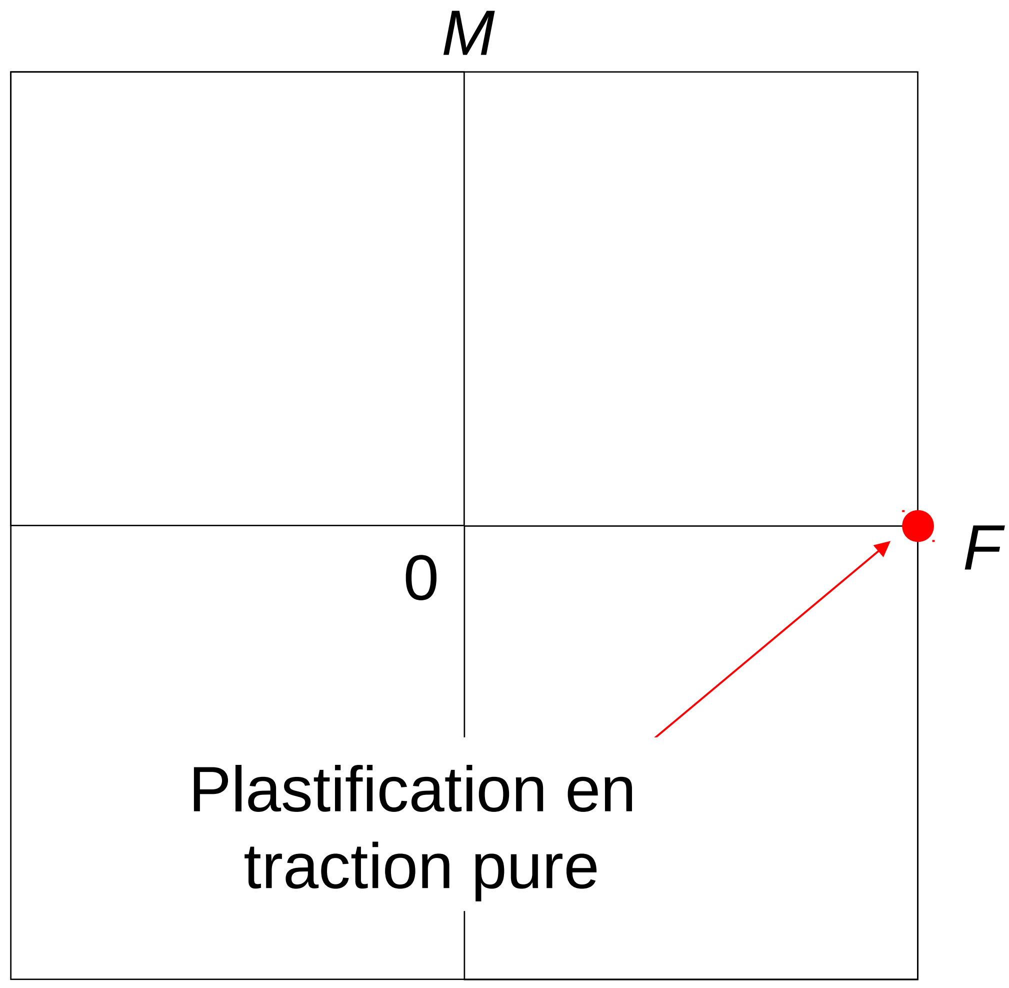




domaine utilisable

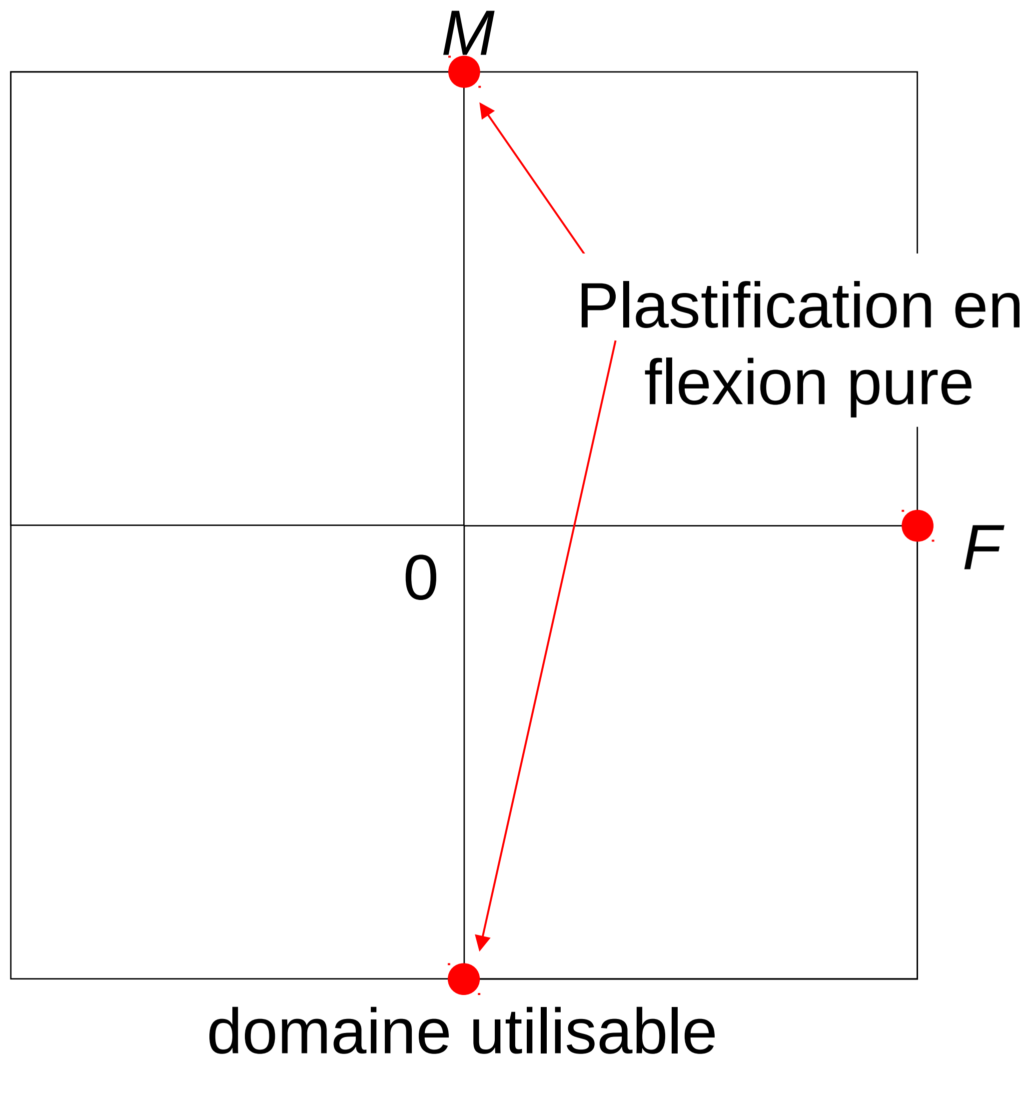
Exemple : domaine limite – poutre console



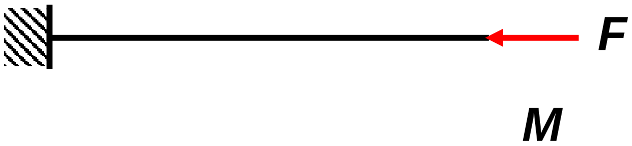


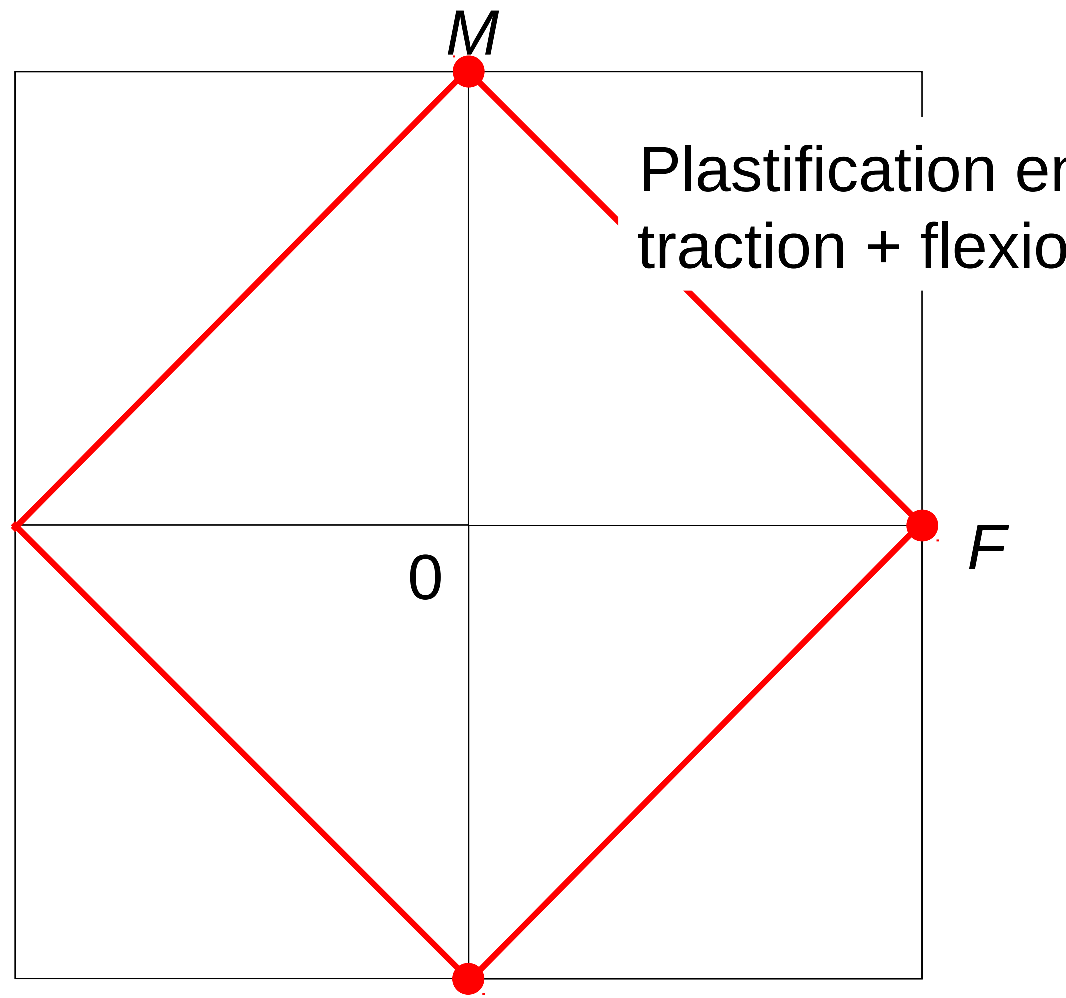
domaine utilisable

Exemple : domaine limite – poutre console

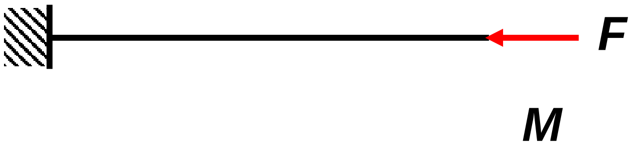


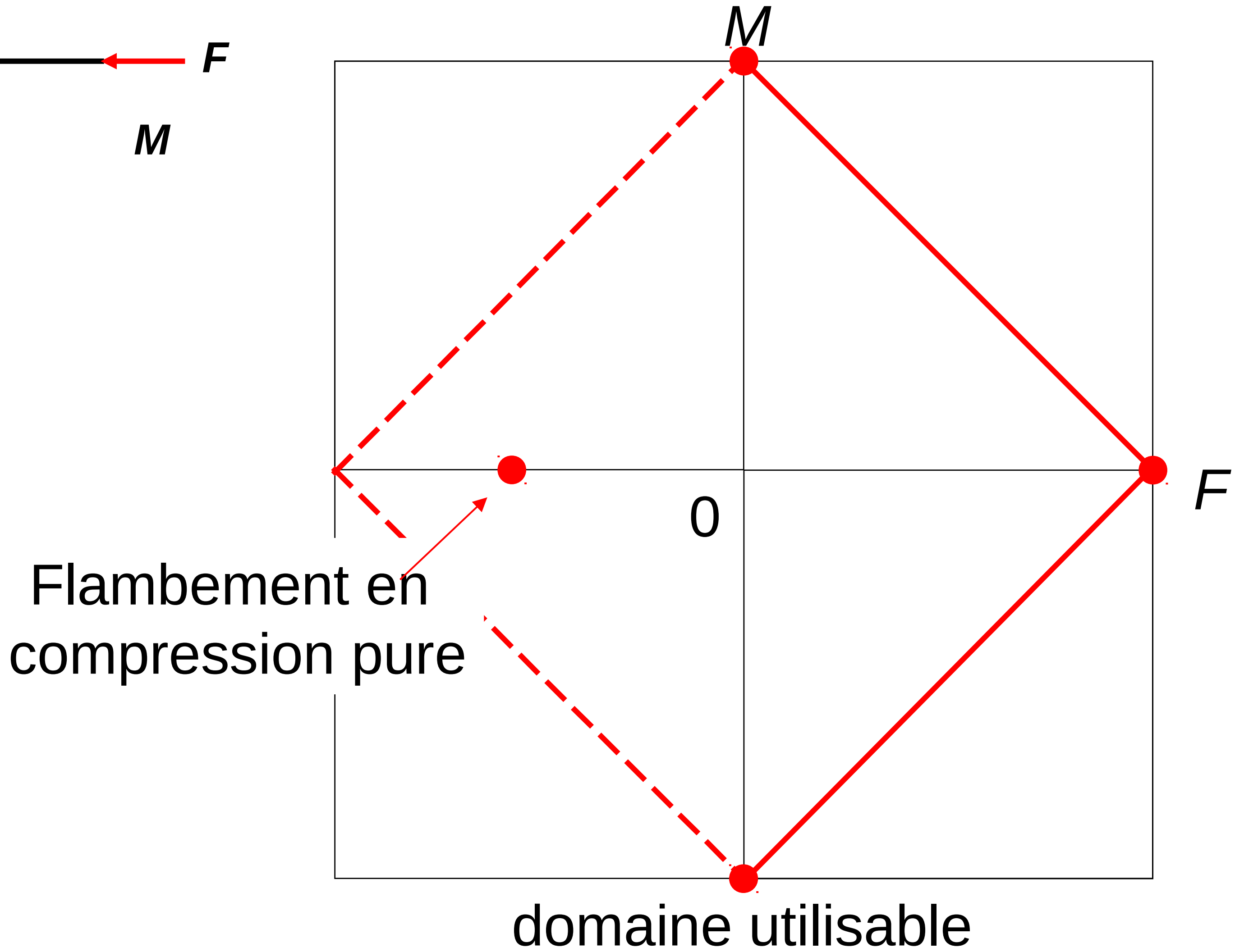
Exemple : domaine limite – poutre console





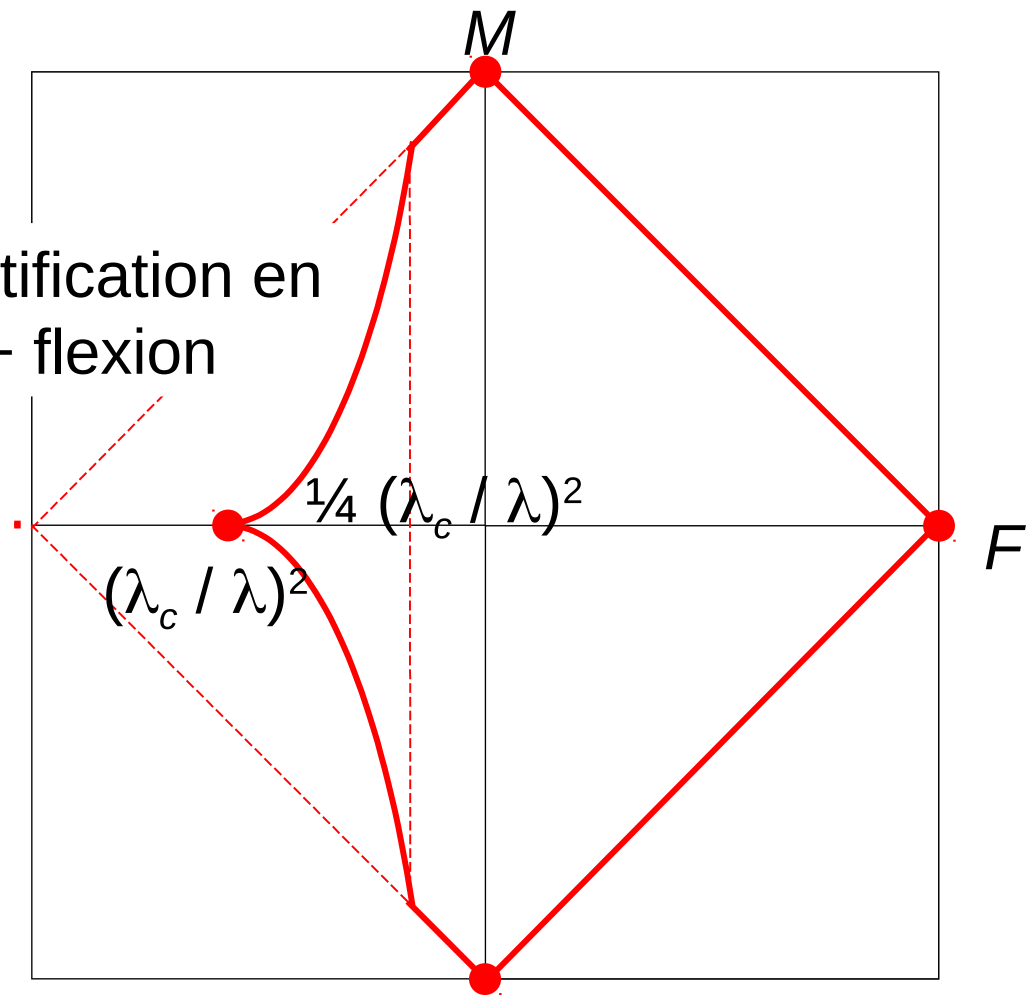
Exemple : domaine limite – poutre console





Exemple : domaine limite – poutre console

Flambement / plastification en compression + flexion

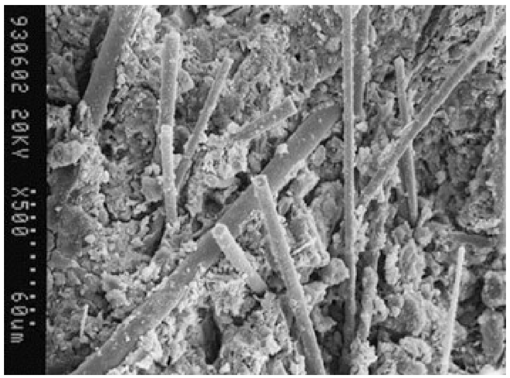


domaine utilisable

Des matériaux

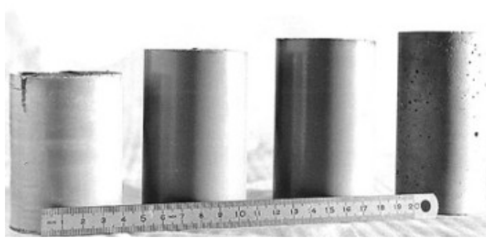
Homogénéité : problème de l'échelle d'observation

60 μm



0,5 cm

10 cm

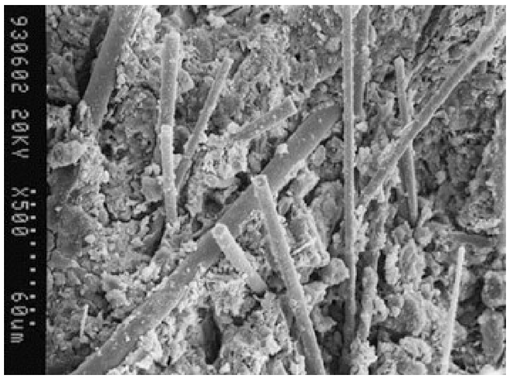


Vues à différentes échelles d'un béton

Des matériaux

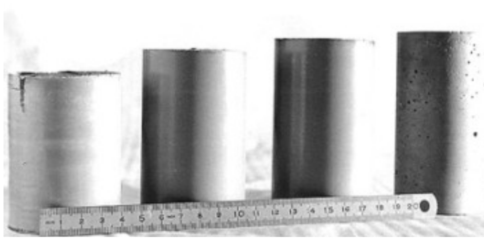
Homogénéité : problème de l’échelle d’observation

60 μm



0,5 cm

10 cm

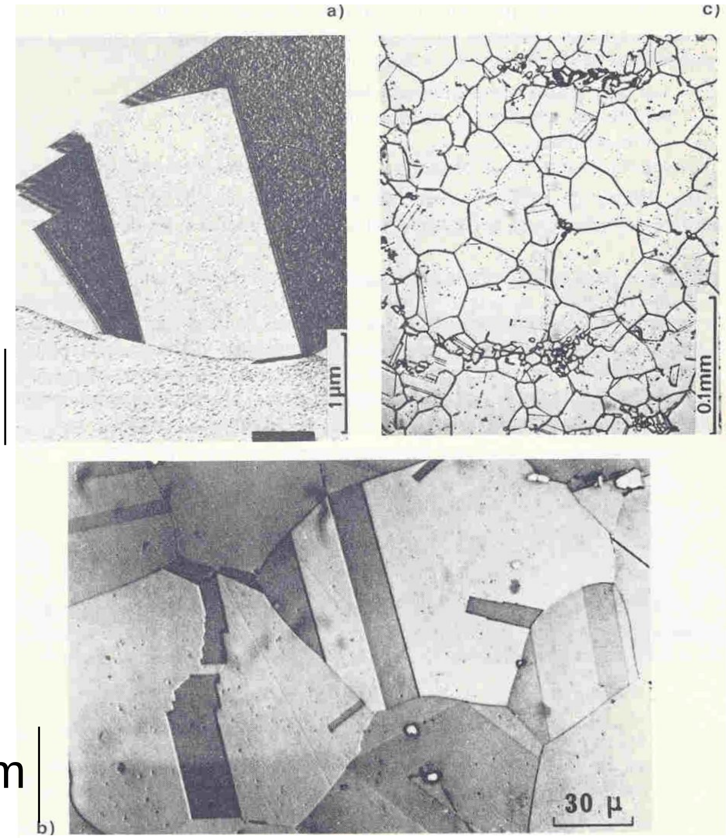


Vues à différentes échelles d’un béton

1 μm

0,1 mm

30 μm



Les alliages métalliques aussi…

source : Lemaître et Chaboche, *Mécanique des Matériaux Solides*, Dunod, 1988

Des matériaux

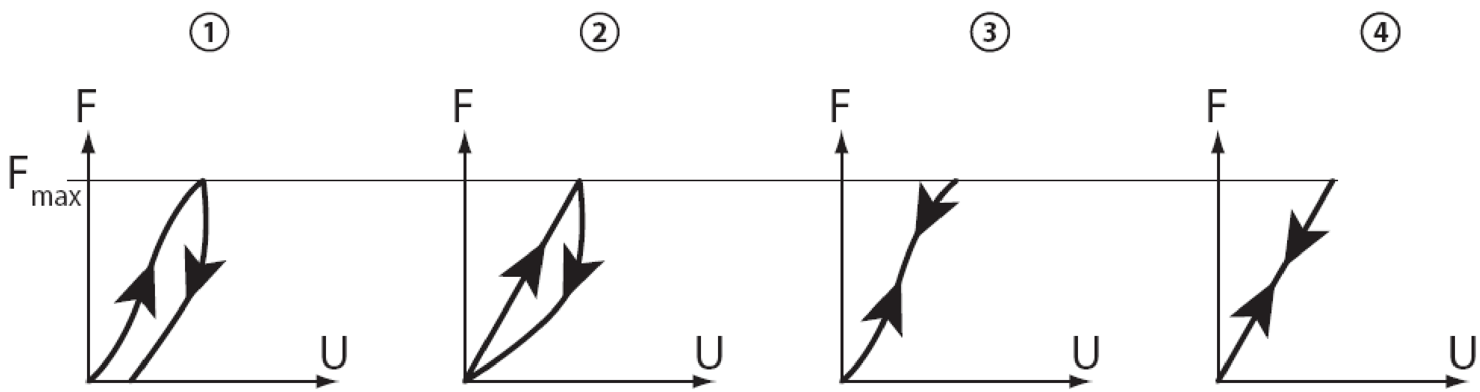
Isotropie

isotrope ⇔ présente les mêmes propriétés dans toutes les directions de l’espace.

Par exemple, dans ce cas, un signal quelconque (son, courant électrique…) peut se propager de la même manière dans toutes les directions.
Si l’on considère un matériau qui possède des fibres ayant une direction privilégiée (comme le bois), du fait de l’orientation particulière de ces fibres, le bois ne sera pas isotrope.

Élasticité linéaire

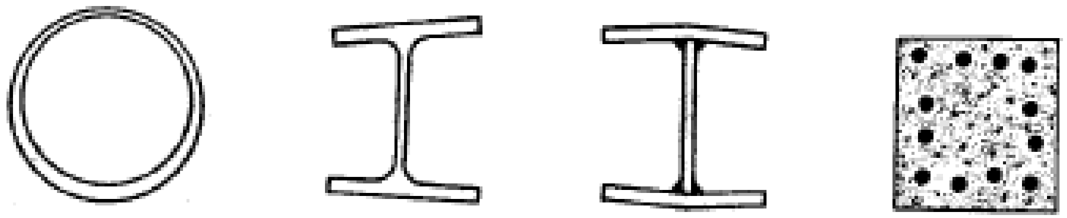
Courbes effort-déplacement pour différents ressorts



~~élastique~~ | ~~élastique~~ | élastique ~~linéaire~~ | élastique linéaire

Des défauts

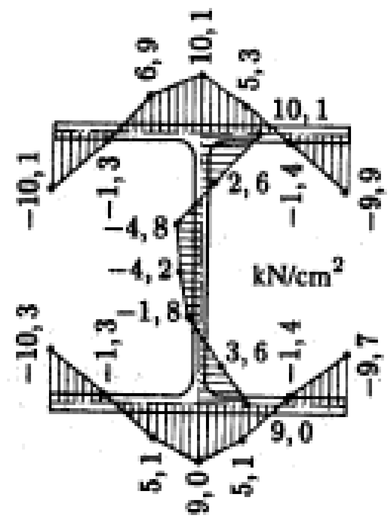
Imperfections géométriques



(a) (b) (c) (d)

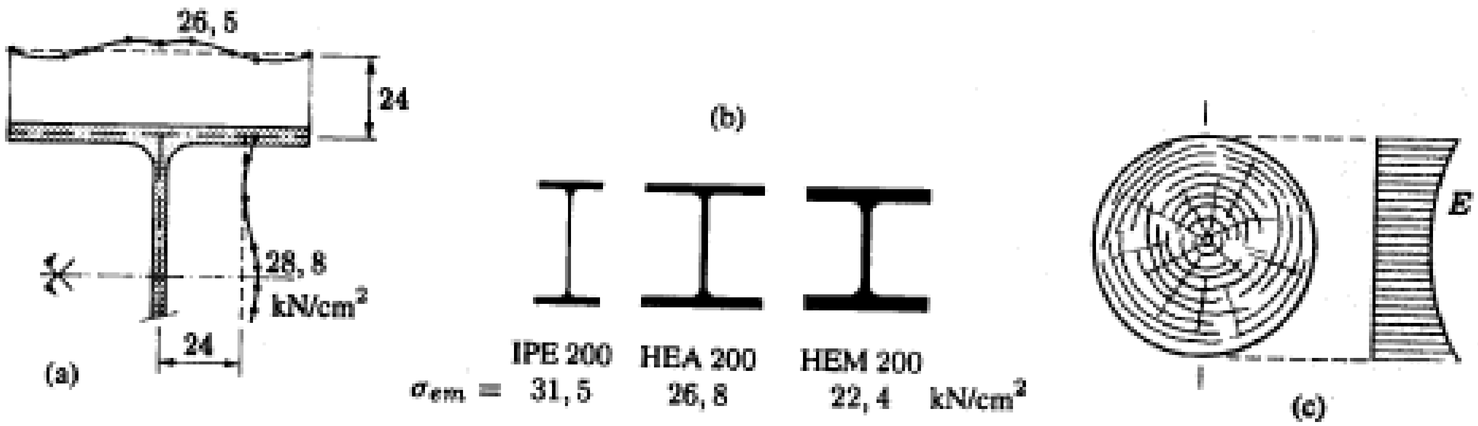
Imperfections géométriques de la section : (a) excentricité des tubes ; (b) tolérances de laminage ; (c) retrait des soudures ; (d) position des armatures du béton armé.

Contraintes résiduelles



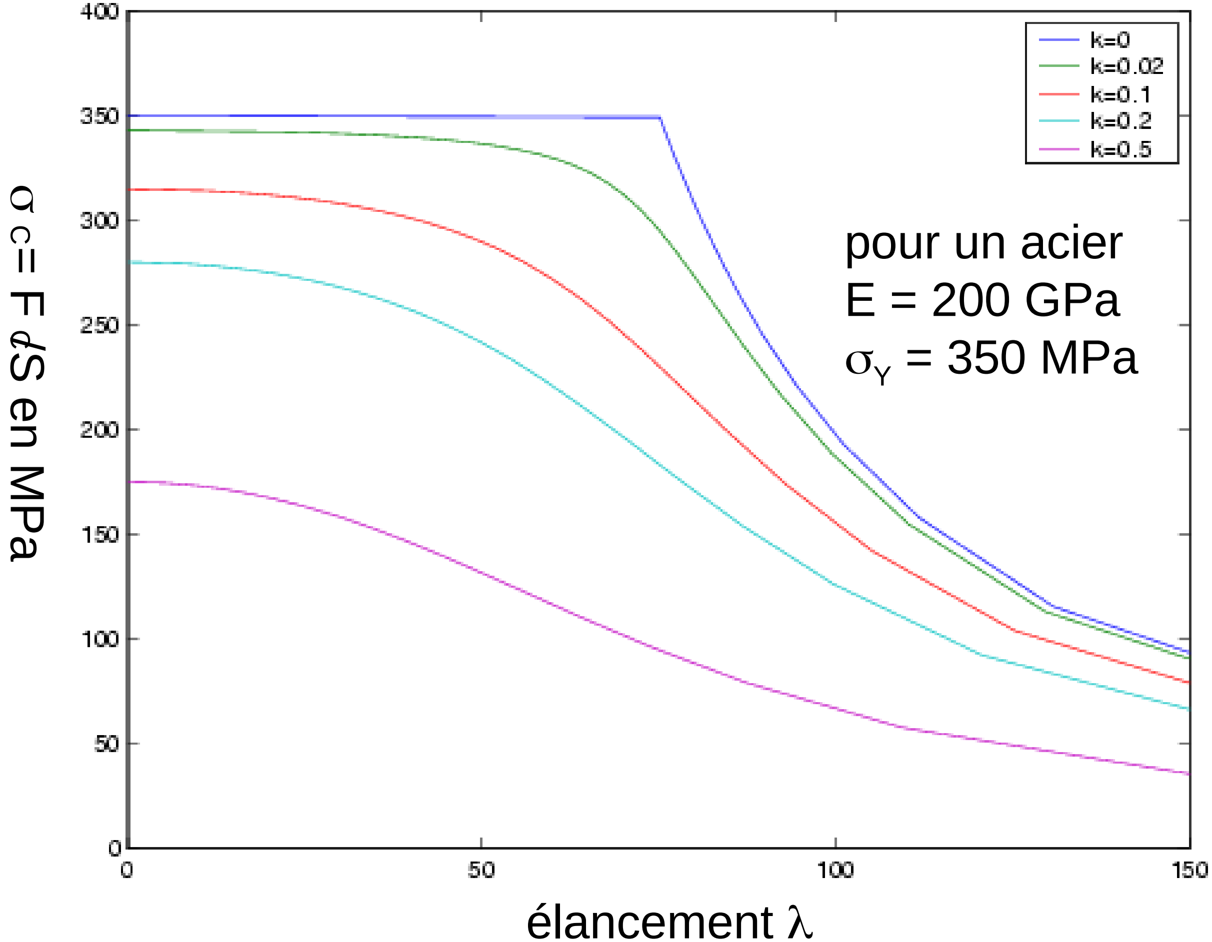
Contraintes normales résiduelles après laminage dans un HEA 200
(contraintes à la ligne moyenne, rabattues dans le plan de la section).

Dispersions des caractéristiques matériau



Imperfections des caractéristiques mécaniques : (a) dispersion de la limite d'élasticité sur un profilé HEA 200 (Fe 360) ; (b) moyenne de la limite d'élasticité dans divers profilés (Fe 360) ; (c) variation du module de Young longitudinal dans la section d'un tronc.

Influence d'imperfections géométriques



$k = \sigma_0 / \sigma_Y$

Opération canette

2 avril 2008 – charge critique mesurée :128 kg – avec défaut : 47 kg

Leonhard EULER

The publication of many articles and his book Mechanica (1736-37), which extensively presented Newtonian dynamics in the form of mathematical analysis for the first time, started Euler on the way to major mathematical work. He integrated Leibniz's differential calculus and Newton's method of fluxions into mathematical analysis. In number theory he stated the prime number theorem and the law of biquadratic reciprocity. Euler made large bounds in modern analytic geometry and trigonometry. He was the most prolific writer of mathematics of all time. His complete works contains 886 books and papers.

Born: 15 April 1707 in Basel, Switzerland
Died: 18 Sept 1783 in St Petersburg, Russia.

source: http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Euler.html

Hermann Ludwig Ferdinand von HELMHOLTZ

Von Helmoltz was a german physicist who first extended Joule's results to a general principle. He expressed the relationship between mechanics, heat, light, electricity, and magnetism by treating them all as manifestations of a single force. He presented a mathematical proof for his Law of Conservation of Force in *Über die Erhaltung der Kraft* (*On the Conservation of Force,* 1847). Helmholtz's use of the word "force" corresponds to what later became known as energy.

Helmholtz sought to synthesize Maxwell's electromagnetic theory of light with the central force theorem. To accomplish this, he formulated an electrodynamic theory of action at a distance in which electric and magnetic forces were propagated instantaneously. He attempted to demonstrate that body heat and muscular action of animals could be produced by the oxidation of foodstuffs.

Born: 31 Aug 1821 in Potsdam, Germany
Died: 8 Sept 1894 in Berlin, Germany.

source: http://scienceworld.wolfram.com/biography/Helmholtz.html

Adhémar Jean Claude BARRÉ de SAINT-VENANT

He was a student at the École Polytechnique, entering the school in 1813 when he was sixteen years old. He graduated in 1816 and spent the next 27 years as a civil engineer. For the first seven of these 27 years Saint-Venant worked for the Service des Poudres et Salpêtres, then he spent the next twenty years working for the Service des Ponts et Chaussées.

Saint-Venant worked mainly on mechanics, elasticity, hydrostatics and hydrodynamics. Perhaps his most remarkable work was that which he published in 1843 in which he gave the correct derivation of the Navier-Stokes equations.

In 1868, Saint-Venant was elected at the mechanics section of the Académie des Sciences. By this time he was 71 years old, but he continued his research and lived for a further 18 years after this time.

Born: 23 Aug 1797 in Villiers-en-Bière, Seine-et-Marne, France.
Died: 6 Jan 1886 in St Ouen, Loir-et-Cher, France.

source: http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Saint-Venant.html

Jacob (Jacques) BERNOULLI

Jacob Bernoulli was a member of the famous family of mathematicians and scientists of Basel, Switzerland. He did important work in connection with elastic curves of beams and was a pioneer in the theory of plate bending and plate vibrations. He was the first to determine that beam curvature is proportional to the bending moment. Bernoulli became famous for his work in theory of probability, analytic geometry, and other fields.

Born: 27 Dec 1654 in Basel, Switzerland
Died: 16 Aug 1705 in Basel, Switzerland.

source: *Mechanics of Materials*, 4th ed., by J.M. Gere and S.P. Timoshenko, PWS Publishing, 1997.

Hooke

Mises