$\mathcal{D}_f =]0;+\infty[\setminus \{-1\} =]0;+\infty[$ 0.5

$$\lim_{0} f(x) = \lim_{0} \frac{\overset{0}{x^2} + \underset{}{\overset{\to -\infty}{\ln(x)}}}{\underset{\to 0}{x+1}} = -\infty.$$

0.5

$\rightarrow$ Asymptote verticale d'équation $x=0$.

$$\lim_{+\infty} f(x) = \lim_{+\infty} \frac{x^2\left(1+\frac{\ln(x)}{x^2}\right)}{x\left(1+1/x\right)} = \lim_{+\infty} x \, \frac{\overset{\to 1}{1+\frac{\ln x}{x^2}}}{\underset{\to 1}{1+1/x}} = +\infty.$$ 0.5

$\Rightarrow$ Recherche d'asymptote oblique

$$m = \lim_{+\infty} \frac{f(x)}{x} = \lim_{+\infty} \frac{1+\frac{\ln x}{x^2}}{1+1/x} = 1$$ $\Rightarrow y = x$ est direction asymptotique 0.5

$$p = \lim_{+\infty} f(x) - mx = \lim_{+\infty} \frac{\cancel{x^2} + \ln x - \cancel{x^2} - x}{x+1} \iff \lim_{+\infty} \frac{-x}{x} = -1 = p$$ 0.5

$\Rightarrow D: y = x-1$ est A.O à $\mathcal{C}_f$ en $+\infty$.

<u>Position relative :</u>

$$\lim_{+\infty} f(x) - y = \lim_{+\infty} \frac{\cancel{x^2} + \ln x - \cancel{x^2} - \cancel{x} + \cancel{x} + 1}{x+1} = 0^+$$ 0.5

$\Rightarrow$ $\mathcal{C}_f$ est au-dessus de D.