$z = 1+i = \sqrt{2}\, e^{i\pi/4}$

$z' = 1+i\sqrt{3} = 2e^{i\pi/3}$

$z \cdot z' = 2\sqrt{2}\, e^{i\left[\frac{\pi}{4}+\frac{\pi}{3}\right]} = 2\sqrt{2}\, e^{i\frac{7\pi}{12}}$

De plus, $z\cdot z' = (1+i)(1+i\sqrt{3}) = 1+i\sqrt{3}+i-\sqrt{3} = 1-\sqrt{3}+i(1+\sqrt{3})$

Par conséquent, $2\sqrt{2}\left[\cos\left(\frac{7\pi}{12}\right)+i\sin\left(\frac{7\pi}{12}\right)\right] = 1-\sqrt{3}+i(1+\sqrt{3})$

$\mathrm{Im}(z\cdot z') = 2\sqrt{2}\sin\left(\frac{7\pi}{12}\right) = 1+\sqrt{3}$

$$\Rightarrow \boxed{\sin\left(\frac{7\pi}{12}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{4}}$$

1

$z = \left(\sqrt{2}e^{-i\pi/4}\right)^8 + \left(2e^{-i\pi/6}\right)^6$

$= (\sqrt{2})^8 e^{-i\frac{8\pi}{4}} + 2^6 e^{-i\frac{6\pi}{6}}$

$= 16\, e^{-i2\pi} + 64 e^{-i\pi} = 16-64 = \boxed{-48 = z}$

1