TD Maths : Primitives et Intégrales #2.

1. Primitives :

$$\int f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)}$$

du type $-\frac{u'(x)}{u(x)}$

$$\hookrightarrow \boxed{F(x) = -\ln|\cos(x)| + C \, ; \; C \in \mathbb{R}}$$

2. Intégrer par parties :

$$\boxed{\int_a^b u'v = [uv]_a^b - \int_a^b uv'}$$

choix de $v(x)$

Méthode ALPES :

① A = Arctan ; arcsin ; arccos
② L = Logarithme
③ P = Polynome
④ E = Exponentielle
⑤ S = Sinus ; cosinus ; tangente

$$I_1 = \int_1^e \frac{\ln(x)}{\sqrt{x}} \, dx .$$

On pose $v = \ln(x)$ donc $u' = \frac{1}{\sqrt{x}}$

alors $u = 2\sqrt{x}$ et $v' = \frac{1}{x}$

donne que : $I_1 = \left[2\sqrt{x}\ln(x)\right]_1^e - 2\int_1^e \frac{\sqrt{x}}{x} \, dx$

et $\begin{cases} \ln(e) = 1 \\ \ln(1) = 0 \end{cases}$

$$\hookrightarrow = -2\int_1^e \frac{dx}{\sqrt{x}} = -2\left[2\sqrt{x}\right]_1^e$$

$$= -4(\sqrt{e} - 1)$$

Alors : $I_1 = 2\sqrt{e} - 4\sqrt{e} + 4 = \boxed{4 - 2\sqrt{e} = I_1}$

$\lim\limits_{x \to 0} \sqrt{x} \ln x = 0$

• $I_2 = \int_0^e \frac{\ln(x)}{\sqrt{x}}\, dx = \left[2\sqrt{x} \ln x\right]_0^e - 4\left[\sqrt{x}\right]_0^e = 2\sqrt{e}\ln e - 0 - 4\sqrt{e}$

$= \boxed{-2\sqrt{e} = I_2}$

3. Intégrer par parties :

1) $I_1 = \int_0^{\pi/4} \frac{x}{\cos^2(x)}\, dx$; on pose $v = x$ donc $u' = \frac{1}{\cos^2 x}$

alors $u = \tan x$ et $v' = 1$

$I_1 = \left[x \tan x\right]_0^{\pi/4} - \int_0^{\pi/4} \tan x\, dx$

$\begin{cases} \tan\left(\frac{\pi}{4}\right) = 1 \\ \tan(0) = 0 \end{cases}$

$\Leftrightarrow \int_0^{\pi/4} \frac{\sin x}{\cos x}\, dx = \left[-\ln|\cos(x)|\right]_0^{\pi/4}$

$= -\ln\left(\frac{\sqrt{2}}{2}\right)$ car $\ln|\cos(0)| = \ln(1) = 0$

$= \boxed{\frac{\pi}{4} + \ln\left(\frac{\sqrt{2}}{2}\right) = I_1}$

2) $I = \int_0^{\pi/3} \frac{x \sin(x)}{\cos^3(x)}\, dx$. Plusieurs choix sont possibles

$v = x$ donc $u' = \frac{1}{\cos^2 x} \cdot \frac{\sin(x)}{\cos(x)}$

$(\tan x)'$ $\qquad$ $\tan(x)$

alors : $v' = 1$ et $u = \frac{1}{2}\tan^2(x)$

$$I = \left[\frac{1}{2} x \tan^2(x)\right]_0^{\pi/3} - \frac{1}{2}\int_0^{\pi/3} \tan^2(x)\, dx$$

$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

$\hookrightarrow \tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1-\cos^2 x}{\cos^2 x} =$

$= \frac{1}{\cos^2 x} - 1$

$$I = \left(\frac{1}{2}\frac{\pi}{3}\cdot(\sqrt{3})^2\right) - \frac{1}{2}\int_0^{\pi/3}\left(\frac{1}{\cos^2 x} - 1\right) dx =$$

$$= \frac{\pi}{2} - \frac{1}{2}\left[\tan x - x\right]_0^{\pi/3} = \boxed{\frac{2\pi}{3} - \frac{\sqrt{3}}{2} = I}$$

3) $I = \int_0^{1/2} (3x^2 - 6x + 1) \cdot \ln(1-x)\, dx$; on pose :

$v = \ln(1-x)$ et $u' = 3x^2 - 6x + 1$

$\Rightarrow v' = \frac{-1}{1-x}$ et $u = x^3 - 3x^2 + x$

$\underline{\text{Alors}}$: $I = \left[(x^3-3x^2+x)\underbrace{\ln(1-x)}\right]_0^{1/2} + \int_0^{1/2} \frac{x^3-3x^2+x}{(1-x)}\,dx$

$\ln\left(\frac{1}{2}\right) = -\ln(2)$

↳ division Euclidienne

$$
\begin{array}{r|l}
x^3 - 3x^2 + x & -x+1 \\
\hline
-(+x^3 - x^2) \quad & -x^2+2x+1 \\
\hline
-2x^2 + x & \\
-(-2x^2+2x) & \\
\hline
-x & \\
-(-x+1) & \\
\hline
-1 &
\end{array}
$$

$$\Rightarrow \frac{x^3-3x^2+x}{1-x} = -x^2+2x+1 - \frac{1}{1-x}$$

↳ $\int = \ln|1-x|$

$$I = \frac{1}{8}\ln(2) + \left[\frac{-x^3}{3} + x^2 + x + \ln|1-x|\right]_0^{1/2}$$

$$I = \frac{1}{8}\ln(2) + \frac{17}{24} + \ln\left(\frac{1}{2}\right) = \boxed{\frac{17}{24} - \frac{7}{8}\ln(2) = I}$$

4) $I = \int_0^{\pi/4} \sin(2x)e^{3x}\,dx$,

Méthode : on pose pour l'IPP 1 : $\begin{cases} u' = e^{3x} & u = \frac{1}{3} e^{3x} \\ v = \sin(2x) \,; & v' = 2\cos(2x) \end{cases}$

$$I_1 = \Big[\underbrace{\frac{1}{3} \sin(2x) e^{3x}}_{= \frac{1}{3} e^{3\pi/4}} \Big]_0^{\pi/4} - \frac{2}{3} \underbrace{\int_0^{\pi/4} \cos(2x) e^{3x} \, dx}_{= I_2}$$

Pour calculer I_2 ; on pose :

$$\begin{cases} u' = e^{3x} & u = \frac{1}{3} e^{3x} \\ v = \cos(2x) & v' = -2\sin(2x) \end{cases}$$

Alors : $I_2 = \Big[\frac{1}{3} \cos(2x) . e^{3x} \Big]_0^{\pi/4} + \frac{2}{3} I$

$= -\frac{1}{3} + \frac{2}{3} I$ car $\cos(\frac{\pi}{2}) = 0$.

Ainsi : $I = \frac{1}{3} e^{\frac{3\pi}{4}} - \frac{2}{3} I_2 = \frac{1}{3} e^{\frac{3\pi}{4}} - \frac{2}{3} \left(-\frac{1}{3} + \frac{2}{3} I \right)$

$$I = \frac{1}{3} e^{\frac{3\pi}{4}} + \frac{2}{9} - \frac{4}{9} I \Rightarrow I \underbrace{\left(1 + \frac{4}{9} \right)}_{13/9} = \frac{1}{3} e^{\frac{3\pi}{4}} + \frac{2}{9}$$

$$\Rightarrow I = \frac{9}{13} \left(\frac{2}{9} + \frac{1}{3} e^{\frac{3\pi}{4}} \right)$$

$$\Rightarrow \boxed{I = \frac{2}{13} + \frac{3}{13} e^{\frac{3\pi}{4}}}$$

4)- . $F(x) = \int \ln(x)\,dx$. On pose : $\begin{cases} u'(x) = 1 \;;\; u(x) = x \\ v(x) = \ln(x) \;;\; v'(x) = \frac{1}{x} \end{cases}$

$$F(x) = x\ln x - \int dx + C = x\ln(x) - x + C \;;\; C \in \mathbb{R}$$

or $F(0) = 0$ et $\lim_{x \to 0} x\ln(x) = 0 \Rightarrow C = 0$

$$\Rightarrow \boxed{F(x) = x\ln(x) - x}$$

. $G(x) = \int x \ln\left(\frac{x}{x+1}\right) dx$. On pose : $\begin{cases} u' = x \\ v = \ln\left(\frac{x}{x+1}\right) \end{cases}$

alors $\begin{cases} u(x) = \frac{x^2}{2} \\ v'(x) = \frac{1}{x(x+1)} \end{cases} \Rightarrow u.v' = \frac{x}{2(x+1)} = \frac{1}{2}\left(1 - \frac{1}{x+1}\right)$

et : $G(x) = \frac{x^2}{2} \ln\left(\frac{x}{x+1}\right) - \frac{1}{2}\int \frac{x}{(x+1)} dx$

$$= \frac{x^2}{2} \ln\left(\frac{x}{x+1}\right) - \frac{1}{2}\int\left(1 - \frac{1}{1+x}\right) dx$$

$$= \frac{x^2}{2} \ln\left(\frac{x}{x+1}\right) - \frac{1}{2}\left(x - \ln(1+x)\right) + C$$

Or $G(1) = 0$:

$$G(1) = 0 = -\frac{1}{2}\cancel{\ln(2)} - \frac{1}{2} + \frac{1}{2}\cancel{\ln}(2) + C \Rightarrow \boxed{C = +\frac{1}{2}}$$

$$\boxed{G(x) = \frac{1}{2}\left(x^2 \ln\left(\frac{x}{x+1}\right) - x + \ln(1+x) + 1\right)}$$

5) Changement de variables :

Méthode : (1) On pose la nouvelle variable

(2) On exprime : $u'(x) =$ afin d'obtenir du en dx

(3) On change les bornes d'intégration.

1) $J = \displaystyle\int_0^2 \frac{dx}{\underbrace{x^2 + 2x + 2}_{(x+1)^2 + 1}}$ avec $u(x) = x + 1$

alors $u'(x) = \dfrac{du}{dx} = 1 \Rightarrow dx = du$

et : $\begin{cases} x_1 = 0 \Rightarrow u_1 = 1 \\ x_2 = 2 \Rightarrow u_2 = 3 \end{cases}$

Alors : $I_1 = \int_1^3 \frac{du}{1+u^2} = \left[\mathrm{Arctan}(u)\right]_1^3 = \boxed{\mathrm{Arctan}(3) - \frac{\pi}{4} = I_1}$

2) $I_2 = \int_{-1}^{+1} \sqrt{1-x^2}\,dx$ avec $x(t) = \sin(t)$

alors $x'(t) = \frac{dx}{dt} = \cos(t) \Rightarrow dx = \cos(t)\,dt$

et : $\begin{cases} x_1 = -1 \Rightarrow t_1 = -\frac{\pi}{2} \\ x_2 = +1 \Rightarrow t_2 = +\frac{\pi}{2} \end{cases}$

Alors $I_2 = \int_{-\pi/2}^{+\pi/2} \sqrt{1-\sin^2(t)}\cdot\cos(t)\,dt =$

$$= \int_{-\pi/2}^{+\pi/2} \sqrt{\cos^2(t)}\cdot\cos(t)\,dt$$

0 sur une période

$$= \int_{-\pi/2}^{+\pi/2} \cos^2(t)\,dt = \int_{-\pi/2}^{+\pi/2} \frac{1+\cos(2t)}{2}\,dt = \frac{1}{2}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = \boxed{\frac{\pi}{2} = I_2}$$

3) $I_3 = \int_1^2 \frac{1}{2+\sqrt{x}}\,dx$ avec $u(x) = \sqrt{x}$

alors $u'(x) = \frac{du}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow dx = 2\sqrt{x}\,du = \boxed{2u\,du = dx}$

et $\begin{cases} x_1 = 1 \Rightarrow u_1 = 1 \\ x_2 = 2 \Rightarrow u_2 = \sqrt{2} \end{cases}$

$$\Rightarrow I_3 = 2\int_1^{\sqrt{2}} \frac{u\,du}{2+u} = 2\int_1^{\sqrt{2}} \frac{(u+2)-2}{2+u}\,du = 2\cdot\int_1^{\sqrt{2}} \left(1 - \frac{2}{2+u}\right)du = 2\left[u - 2\ln|2+u|\right]_1^{\sqrt{2}}$$

$$\boxed{I_3 = 2\left(\sqrt{2} - 2\ln|2+\sqrt{2}| - 1 + 2\ln|3|\right)}$$

4) $I = \int_1^{+\infty} \frac{dx}{x \ln x}$ avec $u(x) = \ln(x)$

alors $u'(x) = \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x\,du$

et $\begin{cases} x_1 = 1 \Rightarrow u_1 = 0 \\ x_2 \to +\infty \Rightarrow u_2 \to +\infty \end{cases}$

Alors : $I = \int_0^{+\infty} \frac{\cancel{x}\,du}{\cancel{x}\,u} = [\ln(u)]_0^{\infty} = \lim_{u \to +\infty} \ln(u) - \lim_{u \to 0} \ln(u) = +\infty$

$= [\ln(\ln x)]_1^{\infty}$

car $\begin{cases} \cdot\ \ln(\ln x)\Big|_{\to +\infty} = +\infty \Rightarrow \frac{\ln(\ln(x))}{x}\Big|_{\to +\infty} = 0 \ ; \text{ BP selon } (Ox) \\ \cdot\ \ln(\ln x)\Big|_{\to 0^+} = -\infty \Rightarrow \frac{\ln(\ln(x))}{x}\Big|_{\to 0^+} = -\infty \ ; \text{ BP selon } (Oy) \end{cases}$

5) $I = \int_0^1 \frac{dx}{\sqrt{4x - x^2}}$ avec $u = \frac{x}{2} - 1 \Rightarrow u'(x) = \frac{du}{dx} = \frac{1}{2} \Rightarrow dx = 2\,du$

et $\begin{cases} x_1 = 0 \Rightarrow u_1 = -1 \\ x_2 = 1 \Rightarrow u_2 = -1/2 \end{cases}$

Alors $I = -\int_{-1/2}^{-1} \frac{\cancel{2}\,du}{\cancel{2}\sqrt{1-u^2}} = -[\operatorname{Arcsin}(u)]_{-1/2}^{-1} = \operatorname{Arcsin}\left(-\frac{1}{2}\right) - \operatorname{Arcsin}(-1)$

$= +\frac{\pi}{2} - \frac{\pi}{6} = \boxed{\frac{\pi}{3} = I}$

car $u^2 = \frac{x^2}{4} - x + 1 \Rightarrow -4u^2 = 4x - x^2 - 4 \Rightarrow 4x - x^2 = 4(1 - u^2)$

ou $x = 2u + 2$ $\Rightarrow -x^2 = -(2+2u)^2 = -(4 + 4u^2 + 8u)$; $\Rightarrow 4x = 8u + 8$ $\Big\}\ 4x - x^2 = -4u^2 + 4 = 4(1 - u^2)$

6) $I = \int \frac{dx}{3 + e^{-x}} = \int \frac{dx}{e^{-x}(3e^x + 1)}$; je pose $u(x) = e^x$

$$\Rightarrow u'(x) = \frac{du}{dx} = e^{x} \Rightarrow dx = e^{-x} du \quad \text{car } \frac{du}{e^{x}} = e^{-x} \cdot du$$

$$I = \int \frac{\cancel{e^{-x}} du}{\cancel{e^{-x}}(3u+1)} = \int \frac{du}{1+3u} = \frac{1}{3} \ln|1+3u| + C = \boxed{\frac{1}{3} \ln|1+3e^{x}| + C \, ; \, C \in \mathbb{R}}$$

7) $$I = \int_0^1 \frac{2x-1}{x^2+x+1} dx = \int_0^1 \frac{(2x+1)dx}{x^2+x+1} - \int_0^1 \frac{2\,dx}{x^2+x+1} = \underbrace{\Big[\ln|x^2+x+1|\Big]_0^1}_{=\ln(3)} - \int_0^1 \frac{2\,dx}{x^2+x+1}$$

Avec:

- $$\frac{2x-1}{x^2+x+1} = \frac{2x+1-2}{x^2+x+1} = \frac{2x+1}{x^2+x+1} - \frac{2}{x^2+x+1}$$

- $$\int_0^1 \frac{2\,dx}{1+x+x^2}$$ je pose $x = \frac{\sqrt{3}u-1}{2} \Rightarrow dx = \frac{\sqrt{3}}{2} du$
  alors $1+x+x^2 = \frac{3}{4}(1+u^2)$ ✳ Voir démo

$$= \frac{\sqrt{3}}{2} \cdot \frac{4}{3} \int_{1/\sqrt{3}}^{3/\sqrt{3}} \frac{2\,du}{1+u^2} = \frac{4\sqrt{3}}{3} \Big[\operatorname{Arctan}(u)\Big]_{\frac{1}{\sqrt{3}}}^{\frac{3}{\sqrt{3}}} = \frac{4\sqrt{3}}{3}\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$$

$$= \frac{4\sqrt{3}}{3}\left(\frac{\pi}{6}\right) = \frac{2\sqrt{3}\,\pi\,\sqrt{3}}{9\,\sqrt{3}} = \frac{2\pi}{3\sqrt{3}}$$

Au final: $\boxed{I = \ln 3 - \frac{2\pi}{3\sqrt{3}}}$

8) $I_8 = \int_{\pi/3}^{\pi/2} \frac{dx}{\sin(x)}$ avec la règle de Bioche $\omega(t) = f(t)dt$

$$\begin{cases} \cdot \text{ Si } \omega(-x) = \omega(x) \Rightarrow u(x) = \cos(x) \\ \cdot \text{ Si } \omega(\pi - x) = \omega(x) \Rightarrow u(x) = \sin(x) \\ \cdot \text{ Si } \omega(\pi + x) = \omega(x) \Rightarrow u(x) = \tan(x) \end{cases}$$

Si deux des trois relations précédentes sont vraies

$$\Rightarrow u(t) = \cos(2x).$$

ici : $\omega(x) = \frac{dx}{\sin(x)} \Rightarrow \omega(-x) = \frac{d(-x)}{\sin(-x)} = \frac{-dx}{-\sin(x)} = \omega(x)$

alors on pose : $u(x) = \cos(x)$ et $du = -\sin x \, dx$

$$\Leftrightarrow dx = -\frac{du}{\sin(x)}$$

$$I_8 = \int_a^b -\frac{du}{\sin^2 x} = -\int_a^b \frac{du}{1-\cos^2(x)} = \int_a^b \frac{-du}{1-u^2}$$

$$= \int_a^b \frac{-1\, du}{(1-u)(1+u)} = -\int_a^b \left(\frac{A_1}{(1-u)} + \frac{B_1}{(1+u)}\right) du = -\int_a^b \left(\frac{1}{2(1-u)} + \frac{1}{2(1+u)}\right) du$$

$$= -\frac{1}{2}\Big[-\ln(1-u) + \ln(1+u)\Big]_{\frac{1}{2}}^{0}$$

car :
$$\begin{cases} \text{si } x_1 = \frac{\pi}{3} & \text{alors } u_1 = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} = a \\ \text{si } x_2 = \frac{\pi}{2} & \text{alors } u_2 = 0 = b \end{cases}$$

Au final :
$$I = -\frac{1}{2}\left(0 + \ln\left(\frac{1}{2}\right) - \ln\left(\frac{3}{2}\right)\right)$$
$$= -\frac{1}{2}\left(-\cancel{\ln 2} + \cancel{\ln(2)} - \ln(3)\right) = \boxed{\frac{\ln(3)}{2} = I}$$

6) Valeur moyenne _ Valeur efficace :

Soit : $f(t) = A\sin(\omega t + \varphi)$

Définitions : Valeur moyenne : $\langle f(t) \rangle = \frac{1}{T}\int_0^T f(t)\,dt$

Valeur efficace : $f_{e\!f\!f}(t) = \frac{1}{T}\int_0^T f^2(t)\,dt$

Valeur moyenne : $\langle f(t) \rangle = \frac{1}{T}\int_0^T A\sin(\omega t + \varphi)\,dt$

$$= \frac{A}{T}\left[-\frac{1}{\omega}\cos(\omega t + \varphi)\right]_0^T = -\frac{A}{\omega T}\underbrace{\left(\cos(2\pi + \varphi) - \cos(\varphi)\right)}_{\text{periodicité du cos !}}$$

$$\Longrightarrow \boxed{\langle f(t) \rangle = 0}$$

"la valeur moyenne d'un sinus sur 1 periode est nulle"

Valeur efficace : $f_{eff}^2(t) = \frac{1}{T}\int_0^T A^2 \sin^2(\omega t + \varphi)\,dt = \frac{A^2}{T}\int_0^T \left(\frac{1-\cos(2(\omega t+\varphi))}{2}\right) dt$

$$= \frac{A^2}{T}\int_0^T \frac{dt}{2} - \frac{A^2}{T}\int_0^T \frac{\cos(2(\omega t+\varphi))}{2}\,dt$$

↳ 2 periodes → = 0.

$$\boxed{f_{eff}^2(t) = \frac{A^2}{2}}$$

" La valeur efficace d'un signal sinusoïdal d'amplitude A est : $\frac{A}{\sqrt{2}}$ "

(*) Démo 17. Méthode 1 :

On cherche à calculer $I_2 = \int_0^2 \frac{2\,dx}{1+x+x^2}$. On souhaite se ramener à la forme $\int \frac{2\,du}{\delta(1+u^2)}$; $\delta \in \mathbb{R}$ est une constante.

On cherche ainsi une relation linéaire entre x et u telle que

$$1 + x + x^2 = \delta(1+u^2)$$

polynômes Deg = 2.

On pose : $x = \alpha u + \beta$; $(\alpha;\beta) \in \mathbb{R}$ relation linéaie entre x et u.

alors $x^2 = (\alpha u + \beta)^2 = \alpha^2 u^2 + 2\alpha\beta u + \beta^2$

et $1 = 1$

ainsi : $1 + x + x^2 = \delta(1+u^2)$

$$\Leftrightarrow 1 + \alpha u + \beta + \alpha^2 u^2 + 2\alpha\beta u + \beta^2 = \delta(1+u^2)$$

$$\Leftrightarrow \alpha^2 u^2 + (2\alpha\beta+\alpha)u + (1+\beta+\beta^2) = \delta + \delta u^2$$

<u>Identification :</u>

(1) $\alpha^2 = \delta$

(2) $2\alpha\beta + \alpha = 0 \Leftrightarrow \alpha(2\beta+1) = 0$ ↗ $\alpha = 0 : \phi$ ↘ $\boxed{\beta = -\frac{1}{2}}$

(3) $1 + \beta + \beta^2 = \delta$

$\hookrightarrow 1 - \frac{1}{2} + \frac{1}{4} = \boxed{\delta = \frac{3}{4}}$

et (1) $\Rightarrow \alpha = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$. (je choisis l'une ou l'autre).

$\hookrightarrow \boxed{\alpha = \frac{\sqrt{3}}{2}}$

Alors le changement de variable s'écrit :

$$\boxed{x = \frac{\sqrt{3}u - 1}{2} \Rightarrow dx = \frac{\sqrt{3}}{2}du \quad \text{et} \quad 1 + x + x^2 = \frac{3}{4}(1+u^2)}$$

<u>Méthode 2 :</u> On cherche $D(x)$ sous la forme $1 + x + x^2 = (x+\alpha)^2 + \beta$

$= x^2 + 2\alpha x + \alpha^2 + \beta$

Alors

$\hookrightarrow 2\alpha = 1 \Rightarrow \boxed{\alpha = \frac{1}{2}}$

$\hookrightarrow \alpha^2 + \beta = 1 \Rightarrow \boxed{\beta = \frac{3}{4}}$

Donc $\frac{\varepsilon}{1+x+x^2} = \frac{\varepsilon}{(x+\frac{1}{2})^2 + \frac{3}{4}}$

$= \frac{4}{3} \cdot \frac{\varepsilon}{1 + \frac{4}{3}(\frac{1}{2}+x)^2} = \frac{4}{3} \cdot \frac{\varepsilon}{1+u^2}$

alors $u^2 = \frac{4}{3}\left(x+\frac{1}{2}\right)^2 \Rightarrow u = \pm\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)$

$\Rightarrow u = \frac{2}{\sqrt{3}}x + \frac{1}{\sqrt{3}} \Rightarrow \boxed{x = \frac{\sqrt{3}u - 1}{2}}$