d'où

$$f(\theta) = \frac{\pi}{2} - \frac{2}{\pi} \sum_{p \geq 0} \frac{\cos(2p+1)\theta}{(2p+1)^2}$$

en $\theta = 0$: $$0 = f(0) = \frac{\pi}{2} - \frac{2}{\pi} \sum_{p \geq 0} \frac{1}{(2p+1)^2}$$

donne $$\boxed{\sum_{p=0}^{\infty} \frac{1}{(2p+1)^2} = \frac{\pi^2}{4}}$$

3) Parseval :

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} \theta^2 \, d\theta = c_0^2 + \sum_{p \in \mathbb{Z}^*} |c_p|^2$$

$$\frac{1}{2\pi} \left[\frac{\theta^3}{3} \right]_{-\pi}^{\pi} = \frac{\pi^2}{4} + \frac{4}{\pi^2} \sum_{p=0}^{\infty} \frac{1}{(2p+1)^4} = \frac{1}{2\pi} \left(\frac{2\pi^3}{3} \right) = \frac{\pi^2}{3}$$

$$\sum_{p=0} \frac{1}{(2p+1)^4} = \left(\frac{\pi^2}{3} - \frac{\pi^2}{4} \right) \frac{\pi^2}{4} = \frac{\pi^4}{48}$$

Ex 1

1) $u_n = \frac{n}{2+n^3} \sim \frac{1}{n^2} \geq 0$

Riemann + equivalents : $\boxed{\sum u_n \text{ CV}}$