Corrigé de CT Janvier

Ex Cours

1] Cf cours.

$$c_0 = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)\,d\theta \underset{\theta \leftarrow \theta-\pi \text{ + périodique}}{=} \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)\,d\theta = \frac{1}{2\pi}\int_{-\pi}^{\pi} |\theta|\,d\theta \underset{|\theta| \text{ paire}}{=} \frac{2}{2\pi}\int_0^{\pi} \theta\,d\theta$$

$$= \frac{1}{\pi}\left[\frac{\theta^2}{2}\right]_0^{\pi} = \frac{\pi}{2} \qquad \boxed{c_0 = \frac{\pi}{2}}$$

2] f est C^1 par m + continue donc (Dirichlet)

$$f(\theta) = S(f)(\theta) = \sum_{p\in\mathbb{Z}} c_p e^{ip\theta} = c_0 + \sum_{p\in\mathbb{Z}^*} c_p e^{ip\theta}$$

~~$= c_0 + 2\sum_{p\geq 1} c_p$~~

$$= c_0 + \sum_{p\geq 1} c_p\left(e^{ip\theta} + e^{-ip\theta}\right) = c_0 + 2\sum_{p\geq 1} \frac{(-1)^p - 1}{\pi p^2}\cos(p\theta)$$

($c_{-p} = c_p$)

$$= c_0 + \sum_{p\geq 0} \frac{-2}{\pi(2p+1)^2}\cos((2p+1)\theta)$$

Rem : $c_{2p} = 0$, $c_{2p+1} = \frac{-2}{\pi(2p+1)^2}$.