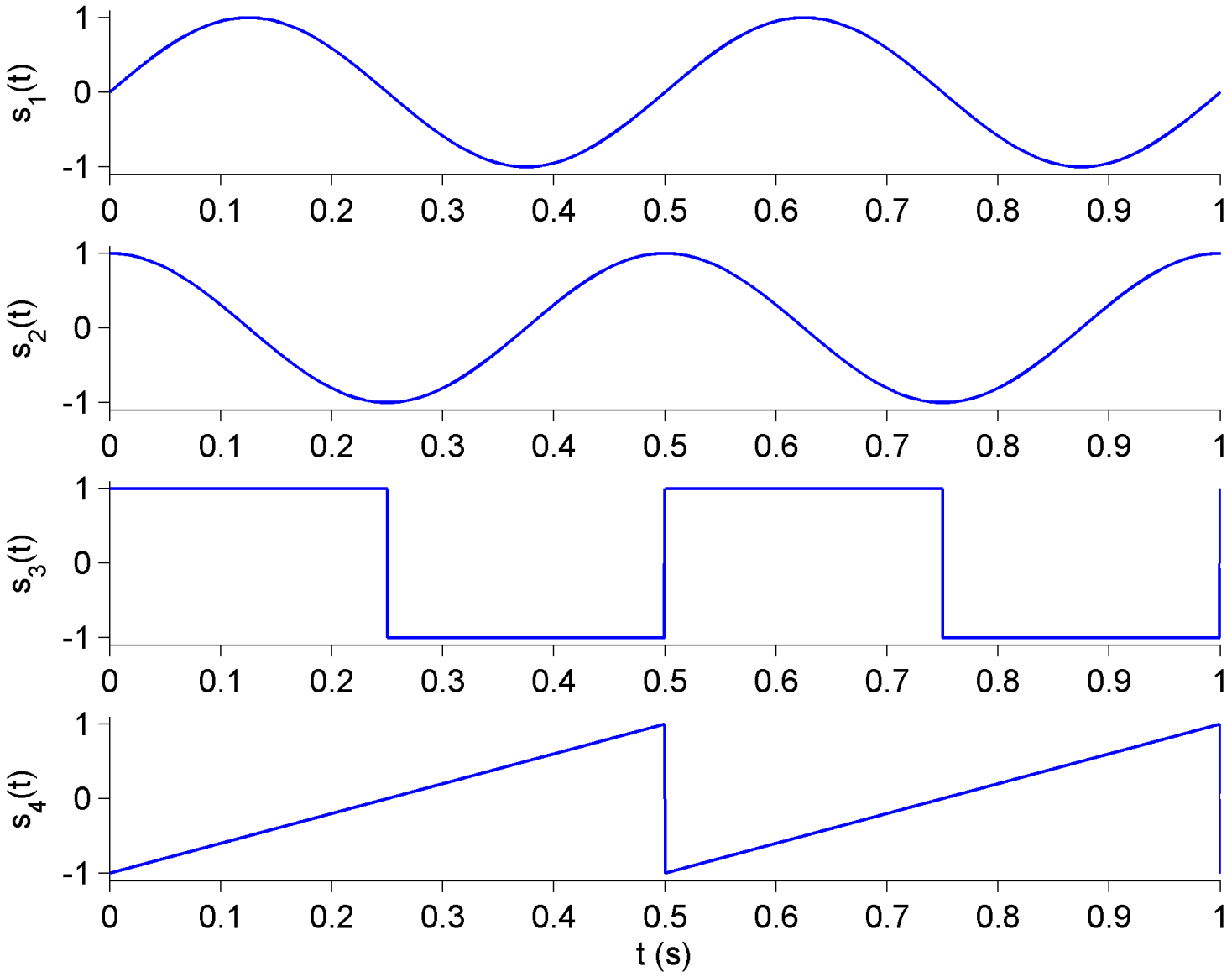
UE1 : Cours de traitement du signal

1ère année du DE d'audioprothésiste
Université Montpellier

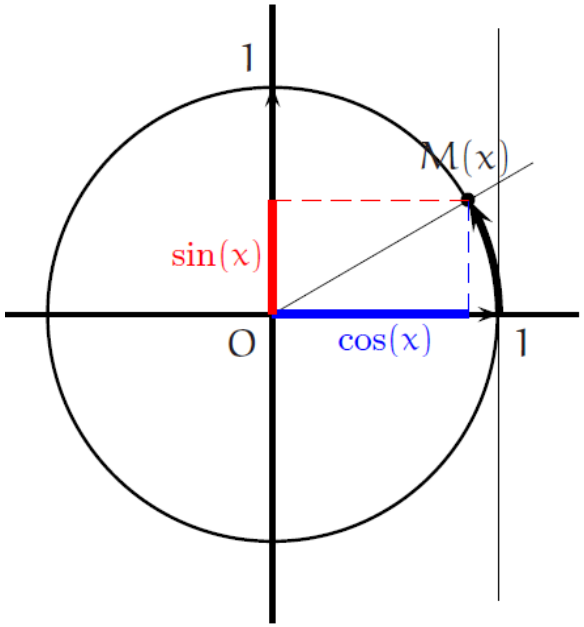
jerome.bourien@umontpellier.fr

Chapitre 1. Les signaux continus *(à temps continu)*

1.1 Les signaux périodiques



Fonctions sinus et cosinus



- M est un point du cercle trigonométrique.

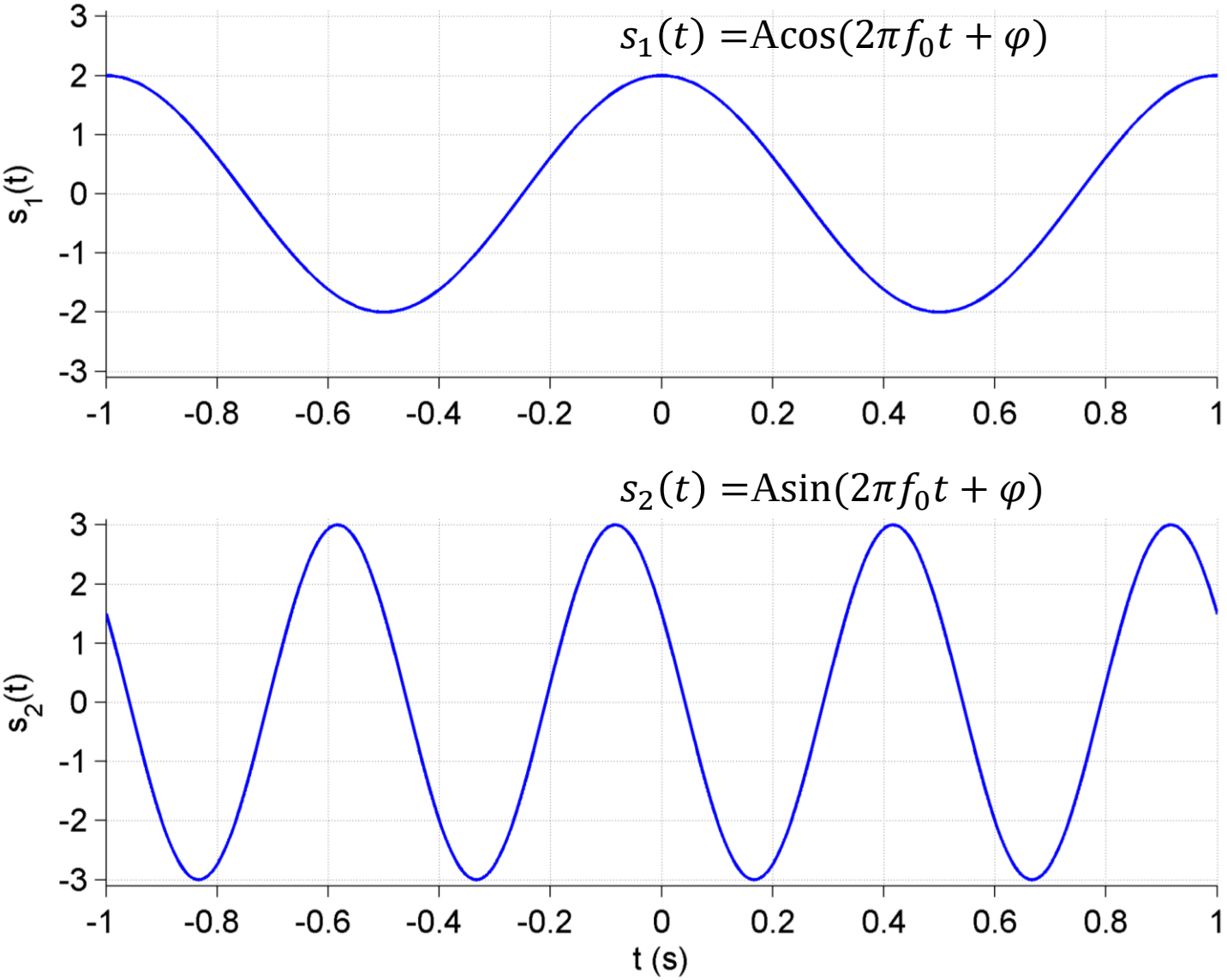
  x est une mesure en radian de l'angle $(\vec{i}, \overrightarrow{OM})$.

  $\cos(x)$ est l'abscisse de M, $\sin(x)$ est l'ordonnée de M.

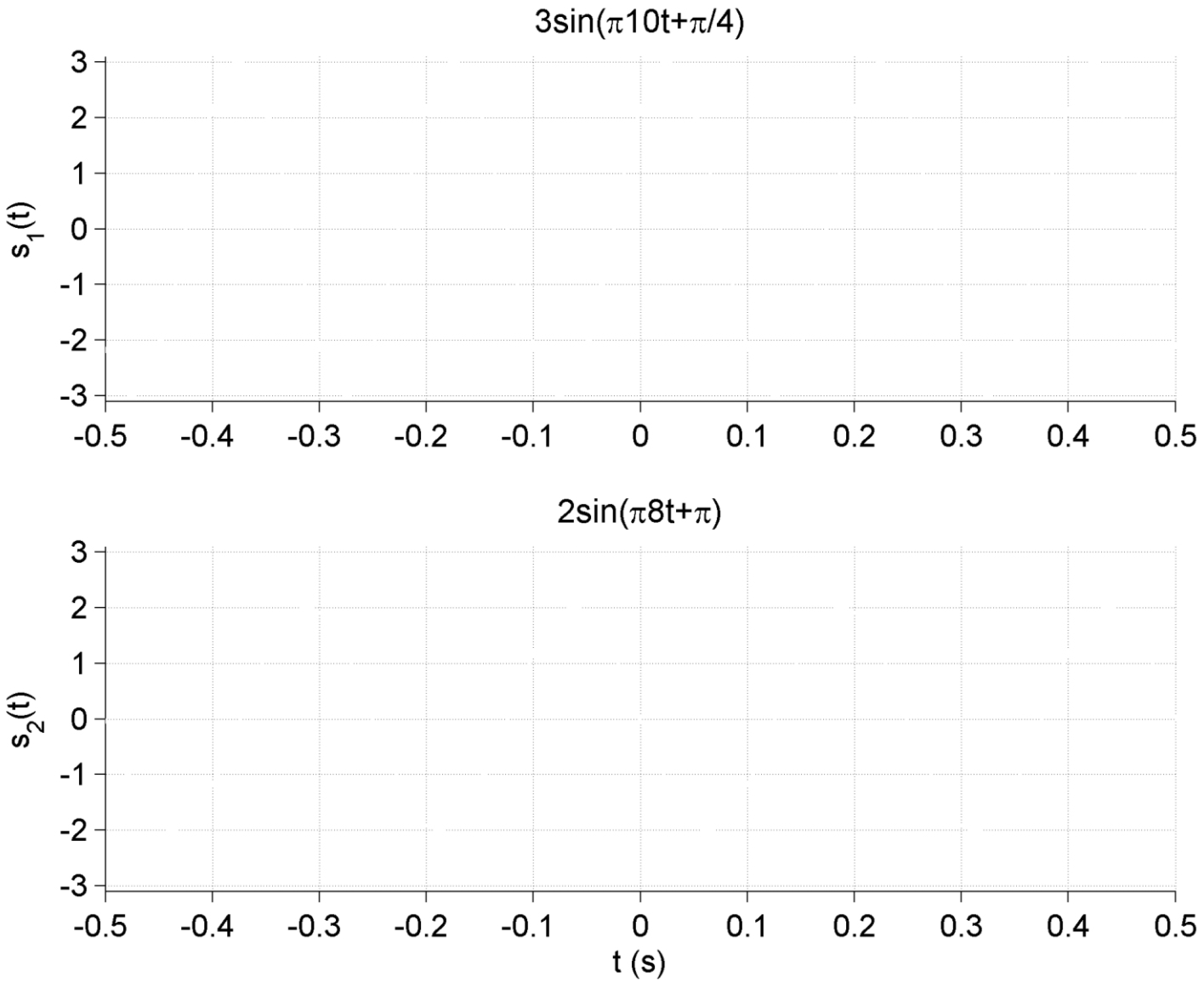
- Pour tout réel x, $\cos^2(x) + \sin^2(x) = 1$.

(Voir formulaire de trigonométrie ci-joint)

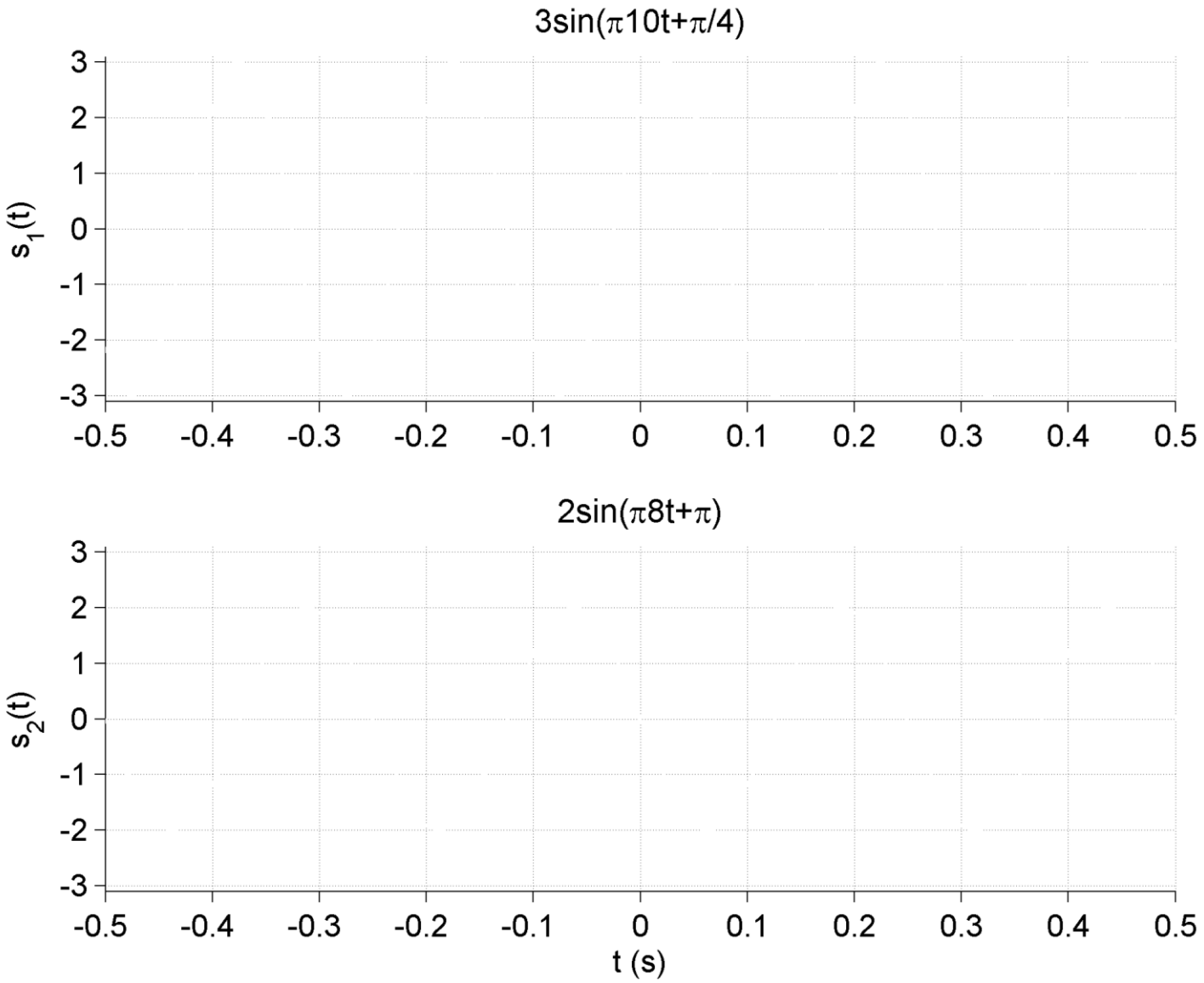
Exercice: Déterminez les paramètres des signaux suivants:



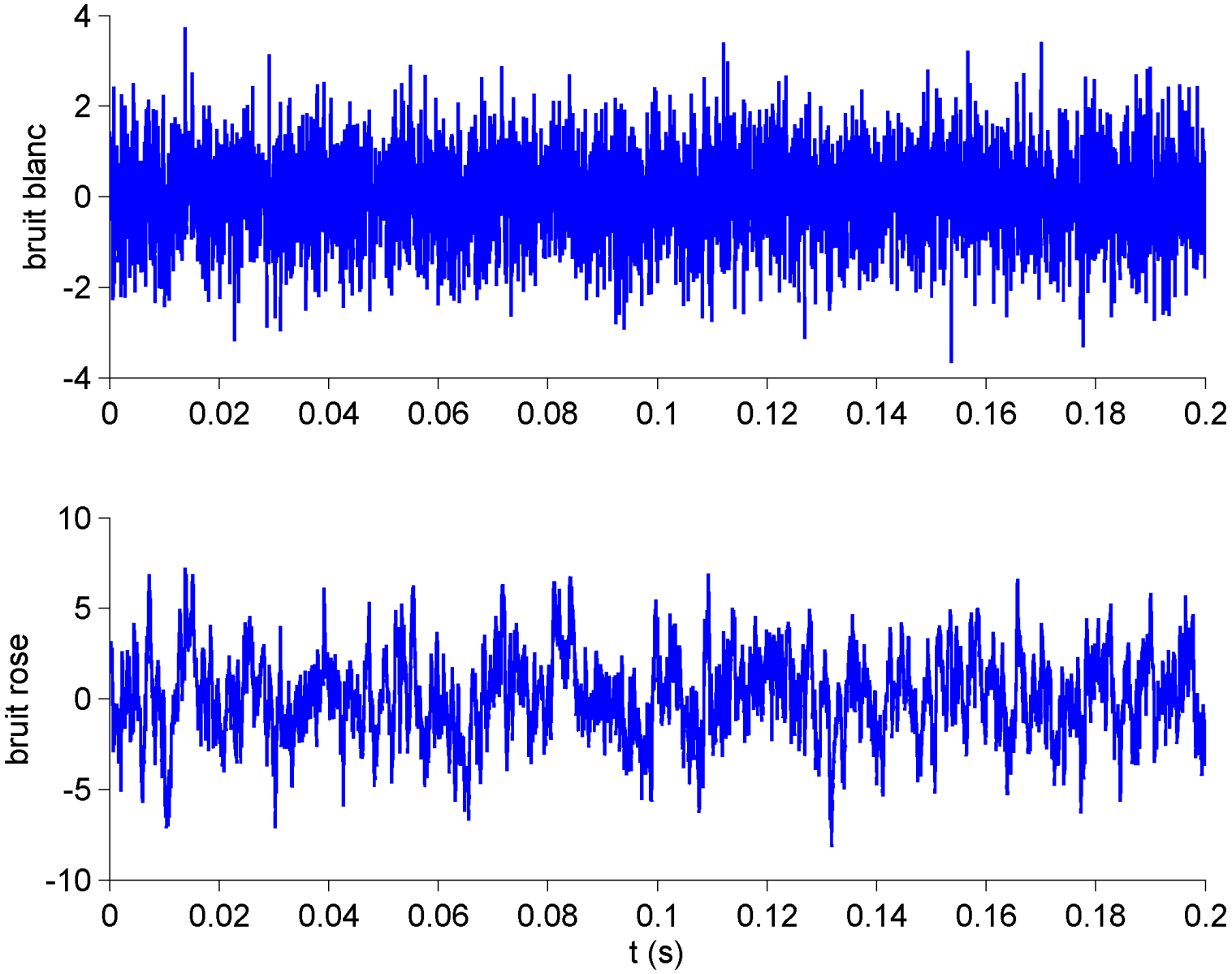
Exercice: Tracez les signaux suivants



Exercice: Tracez les signaux suivants

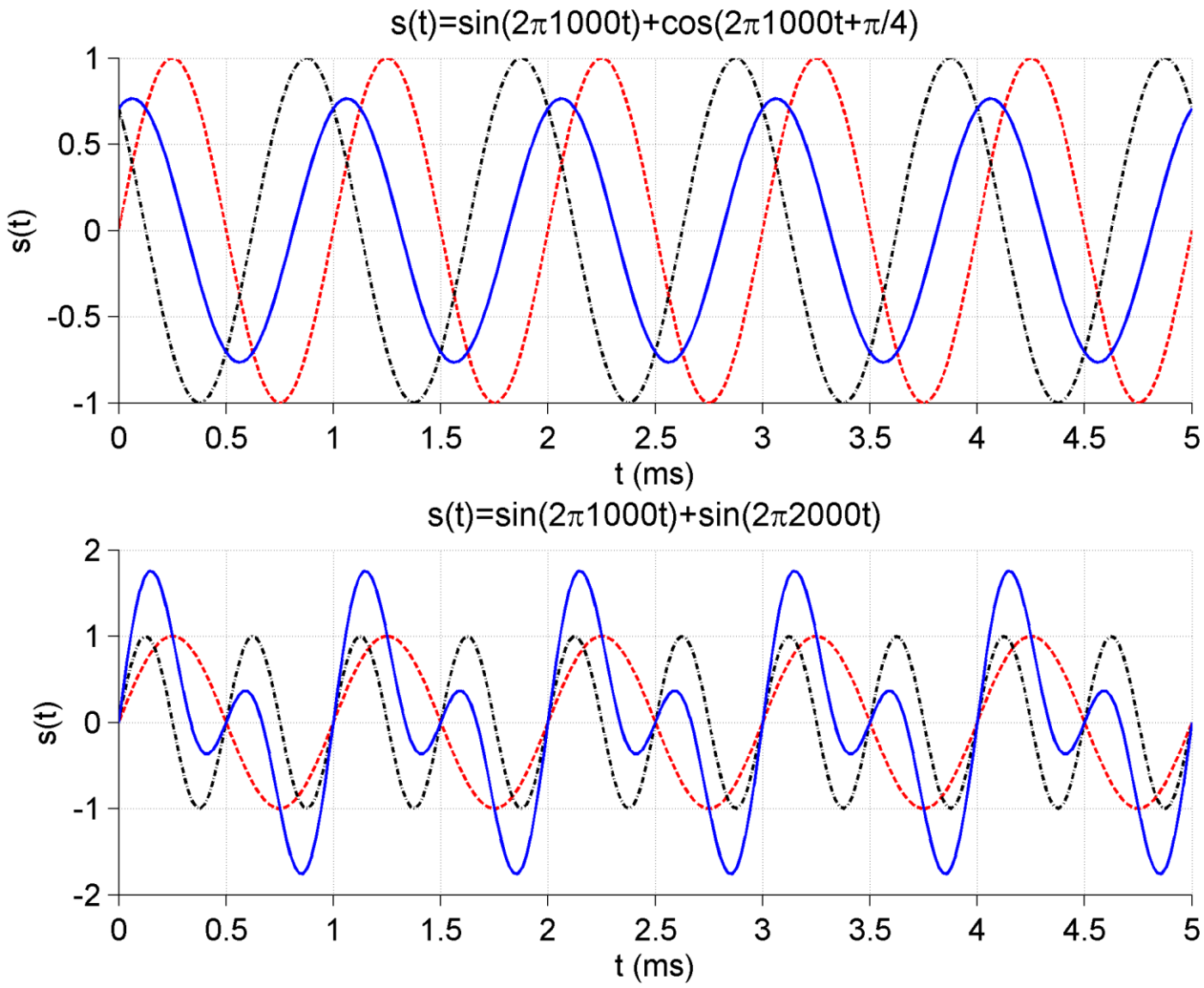


1.3 Les signaux aléatoires

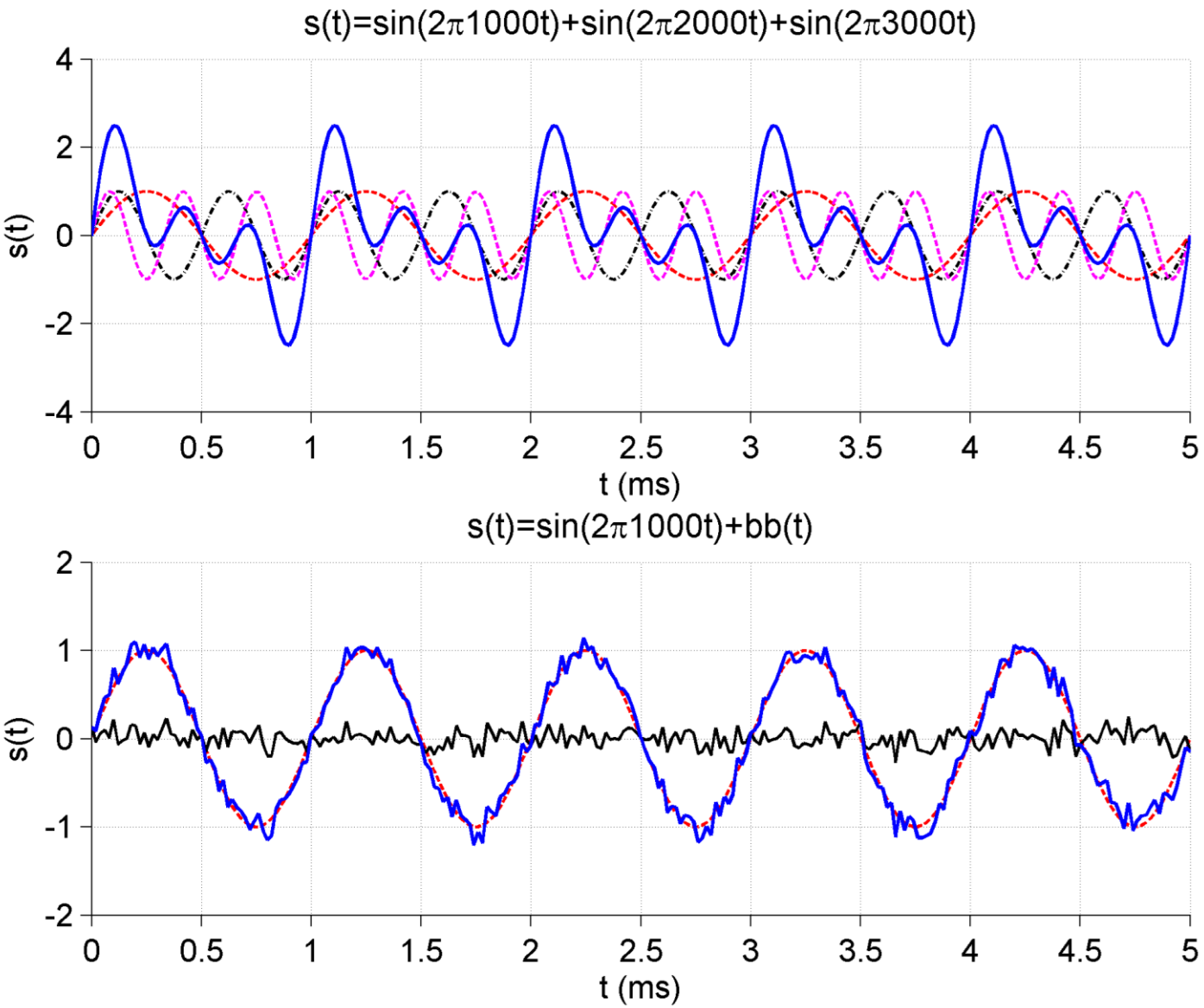


Chapitre 2. Manipulation des signaux

2.1 L'addition

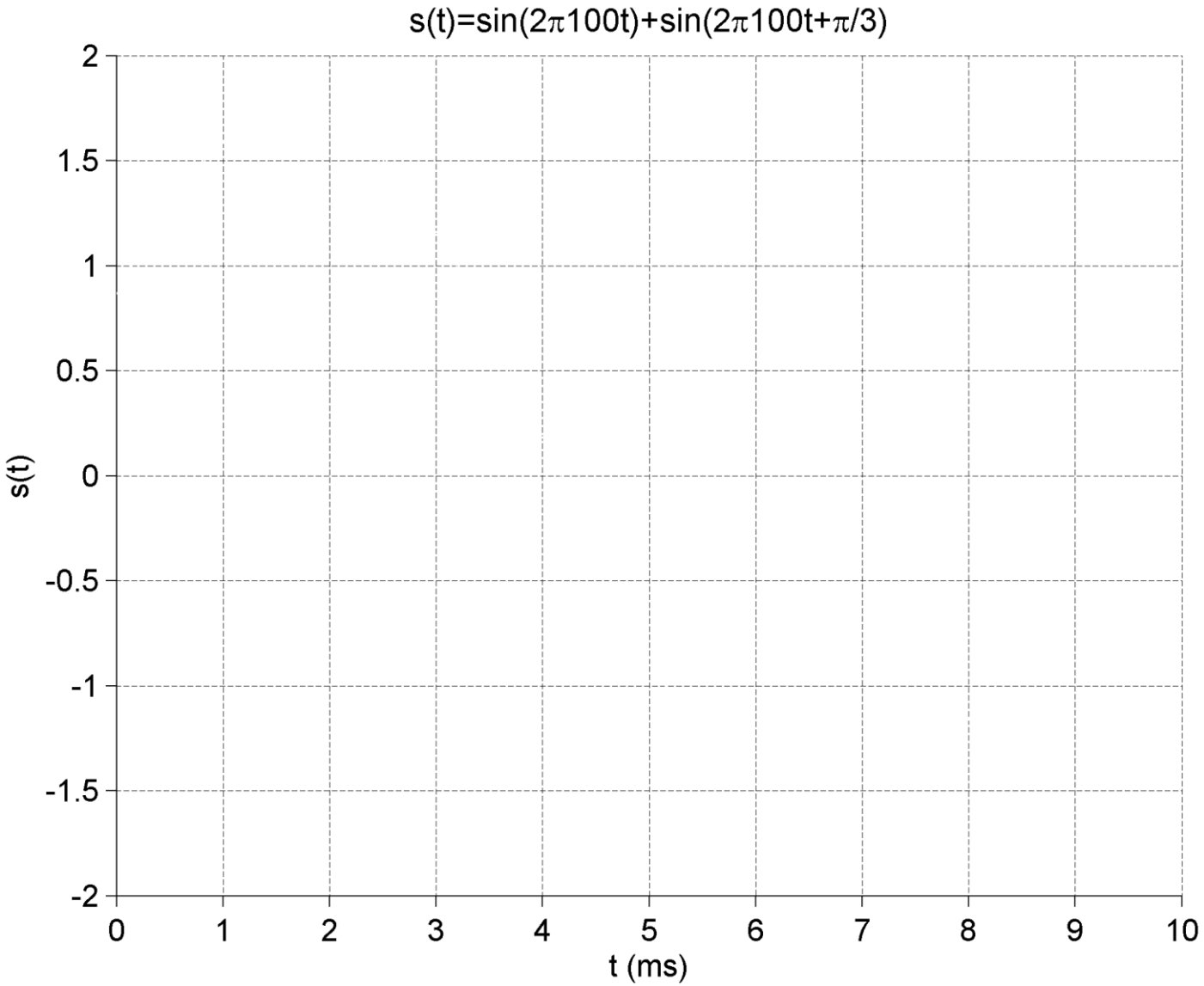


2.1 L'addition

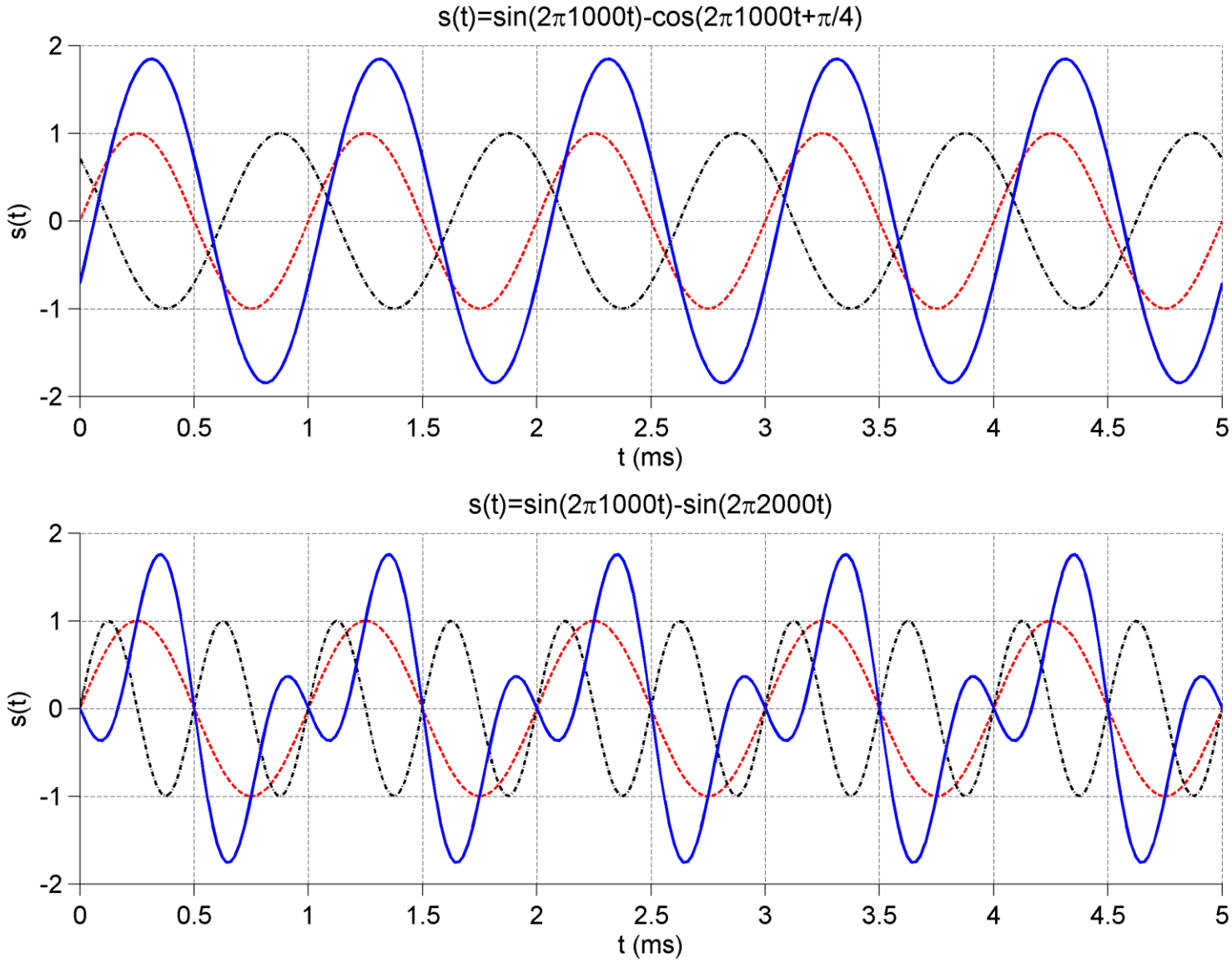


2.1 L'addition : formule utile $sin(a) + sin(b) = 2sin(\frac{a+b}{2})cos(\frac{a-b}{2})$

Tracez:



2.2 La soustraction

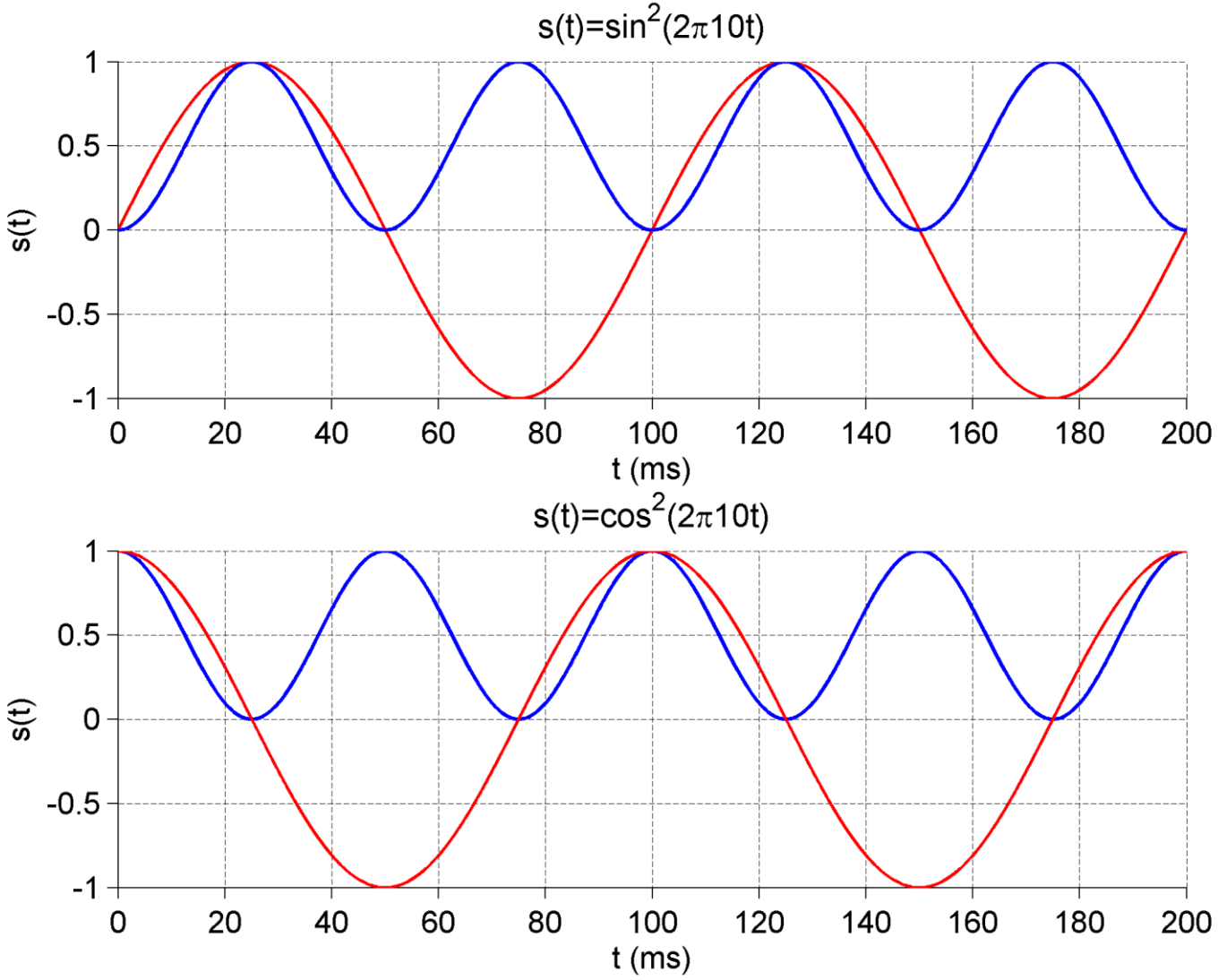


2.2 La soustraction

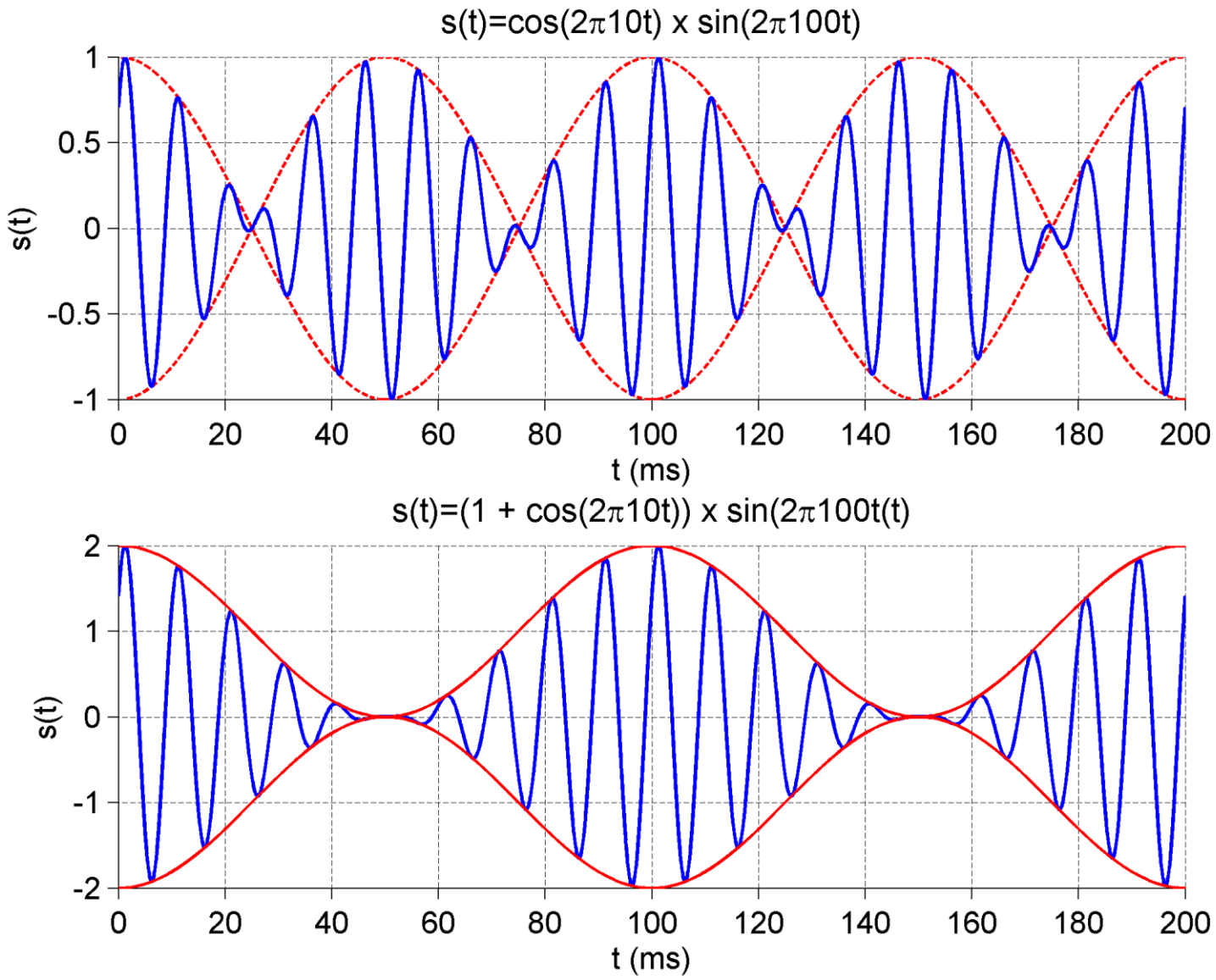
En utilisant la formule donnée en page 10, calculez:

$$sin(a) - sin(b) =$$

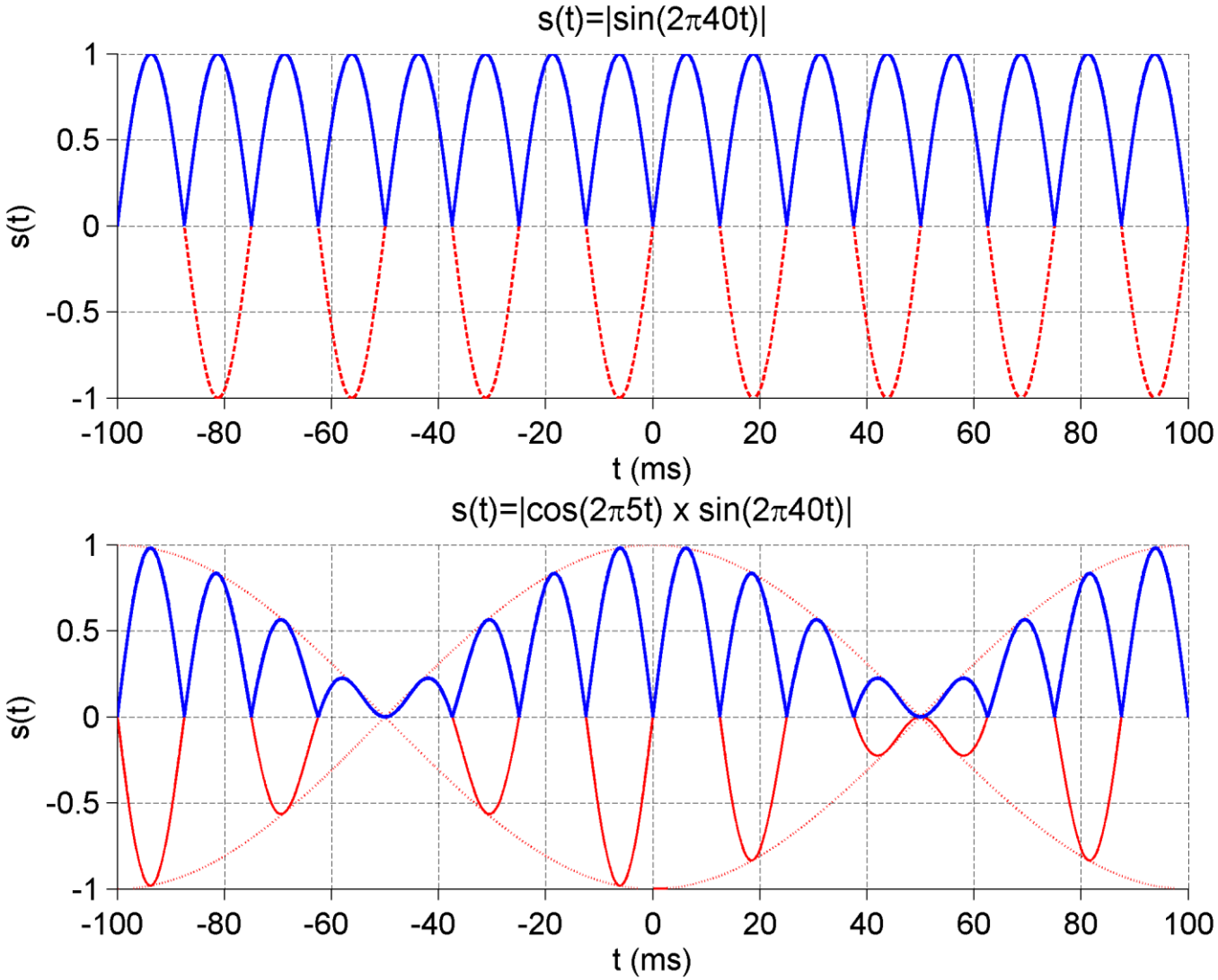
2.3 La multiplication



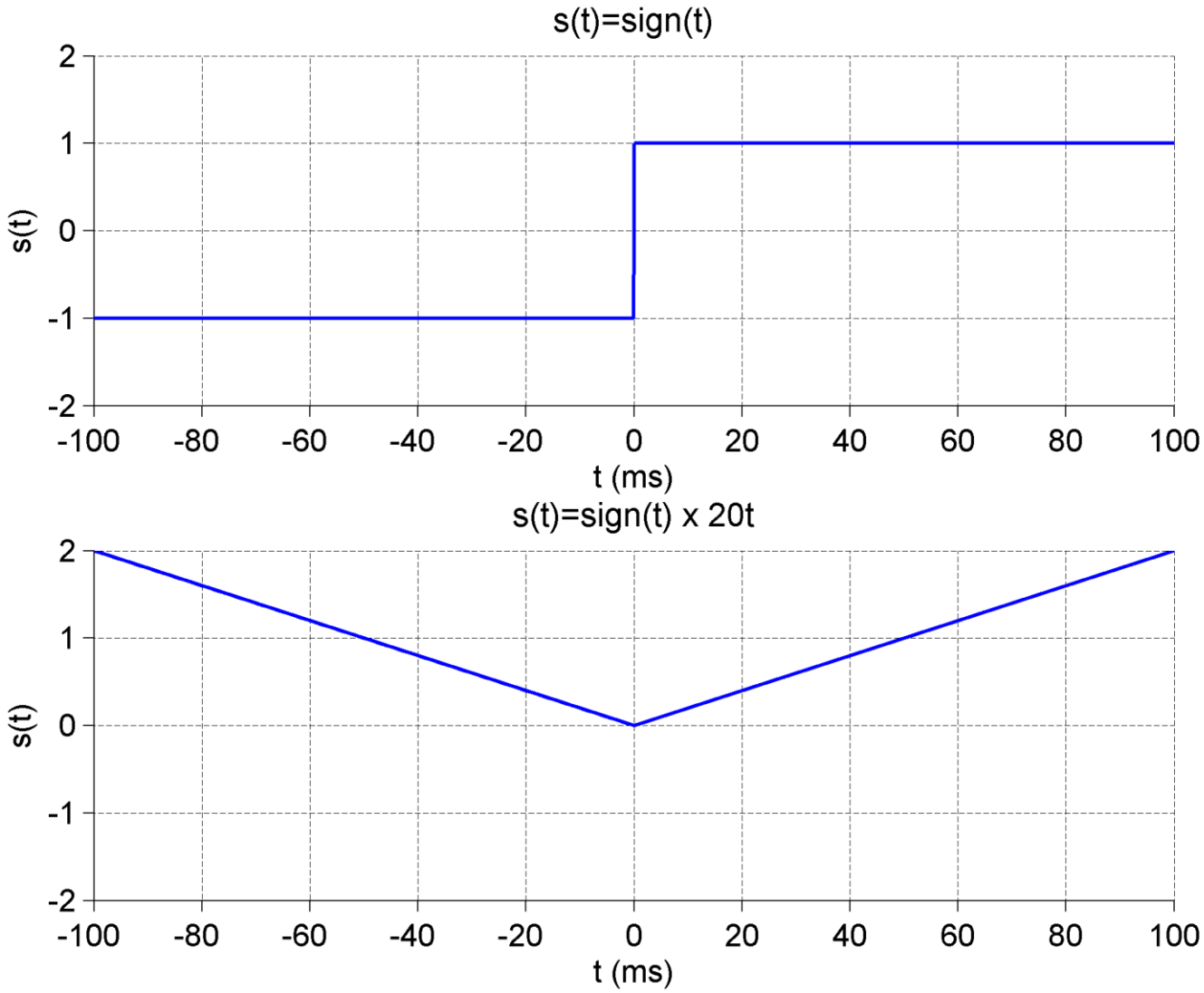
2.3 La multiplication



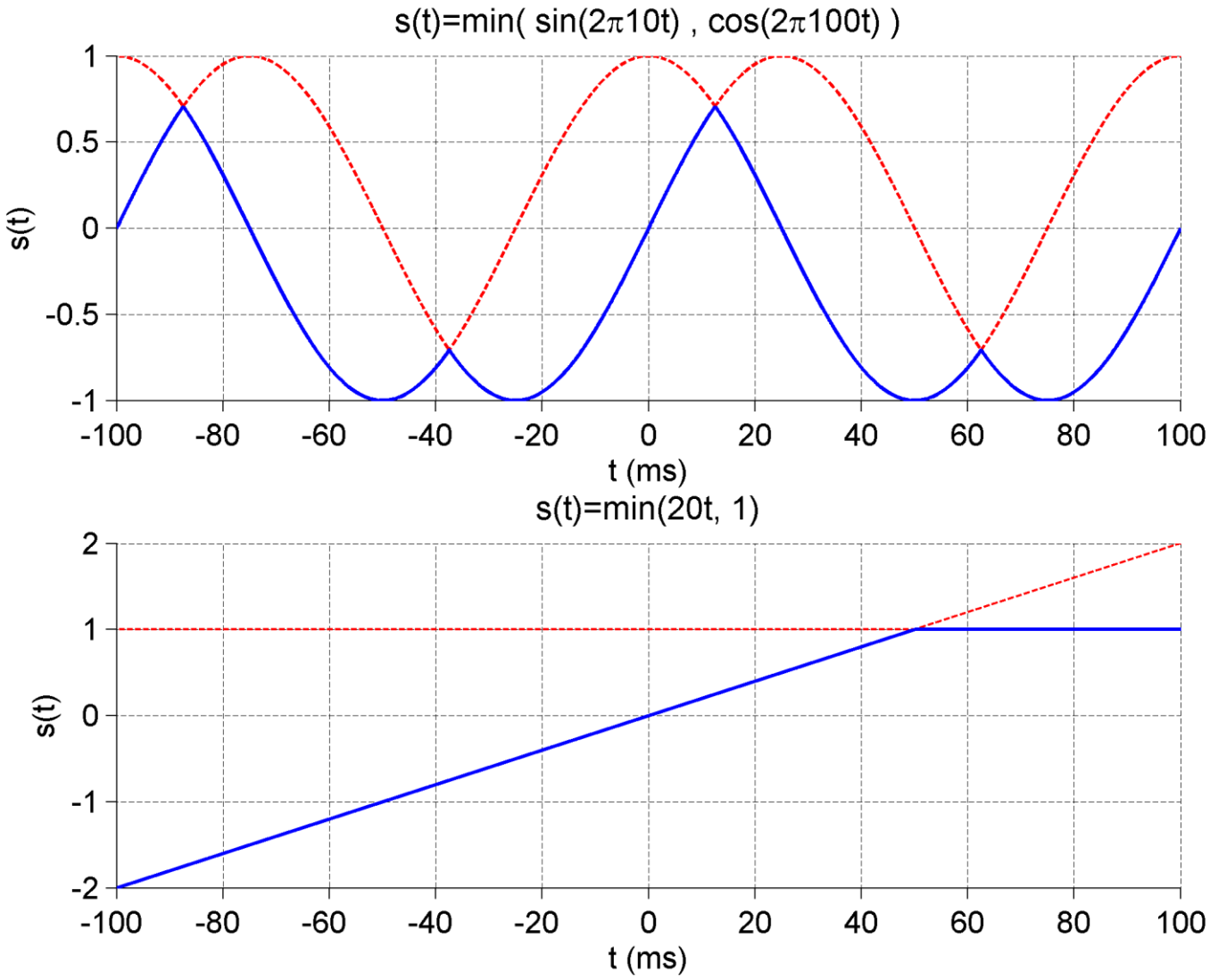
2.5 La fonction « valeur absolue »:



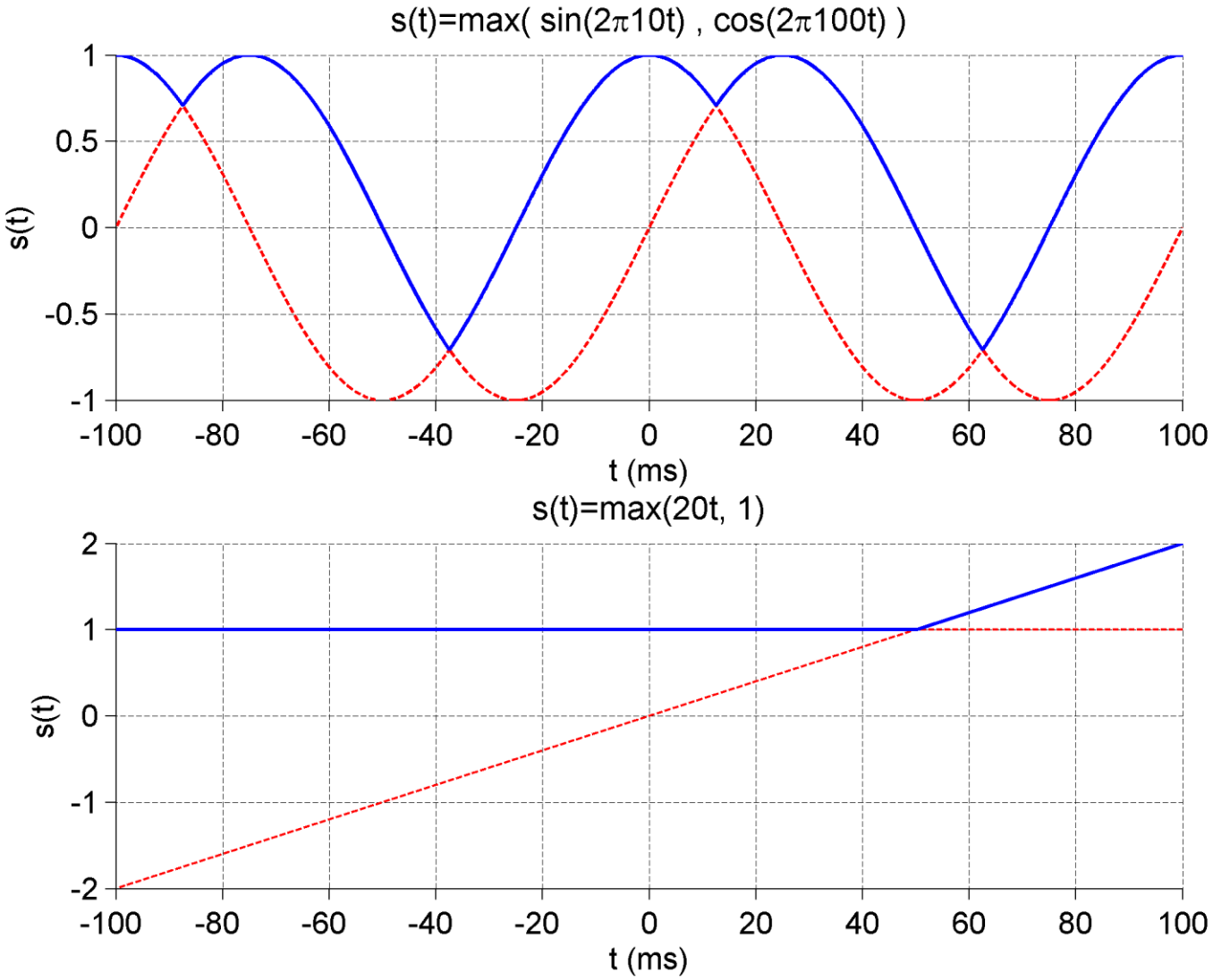
2.6 La fonction « sign »:



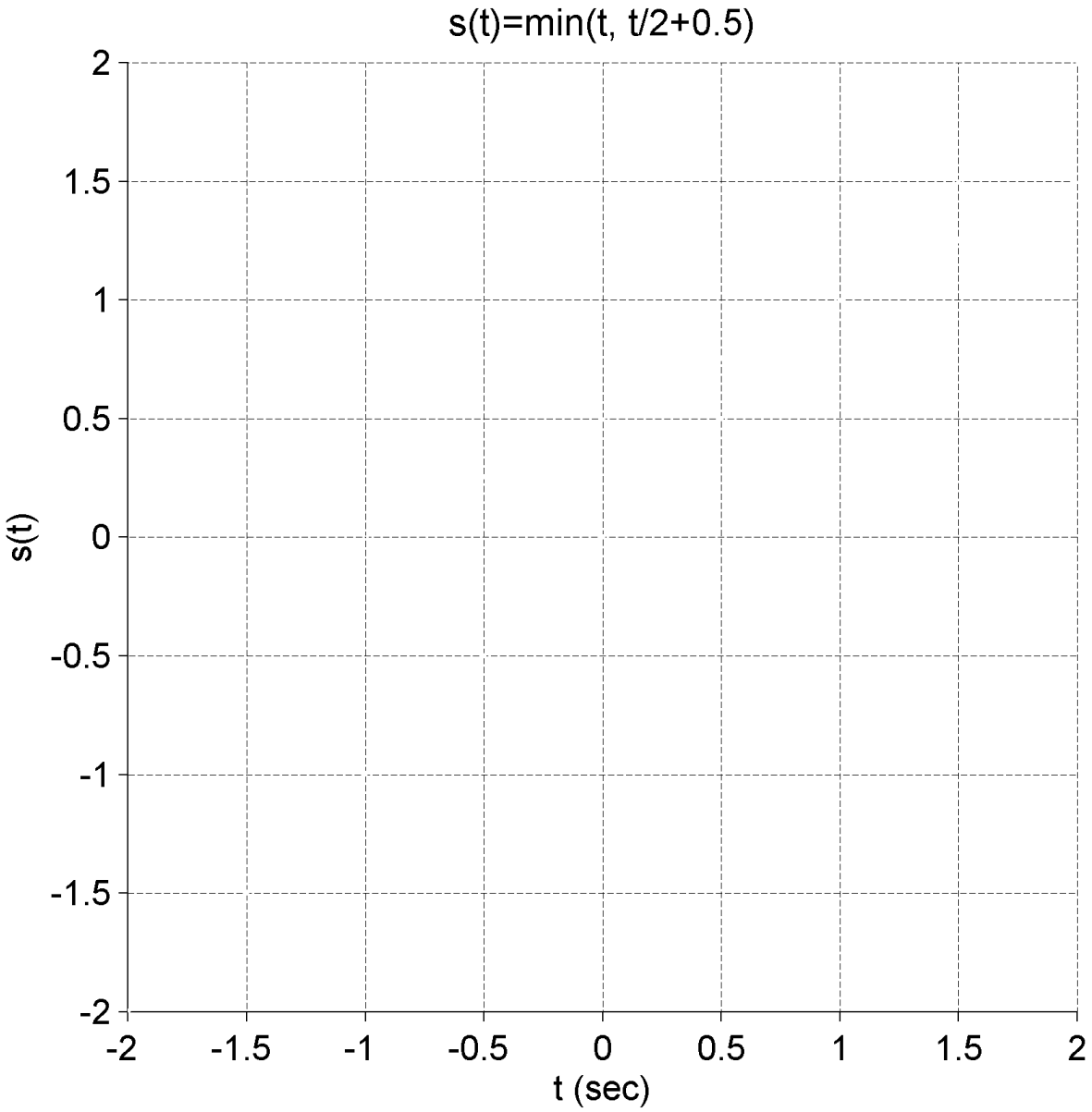
2.7 La fonction « min »



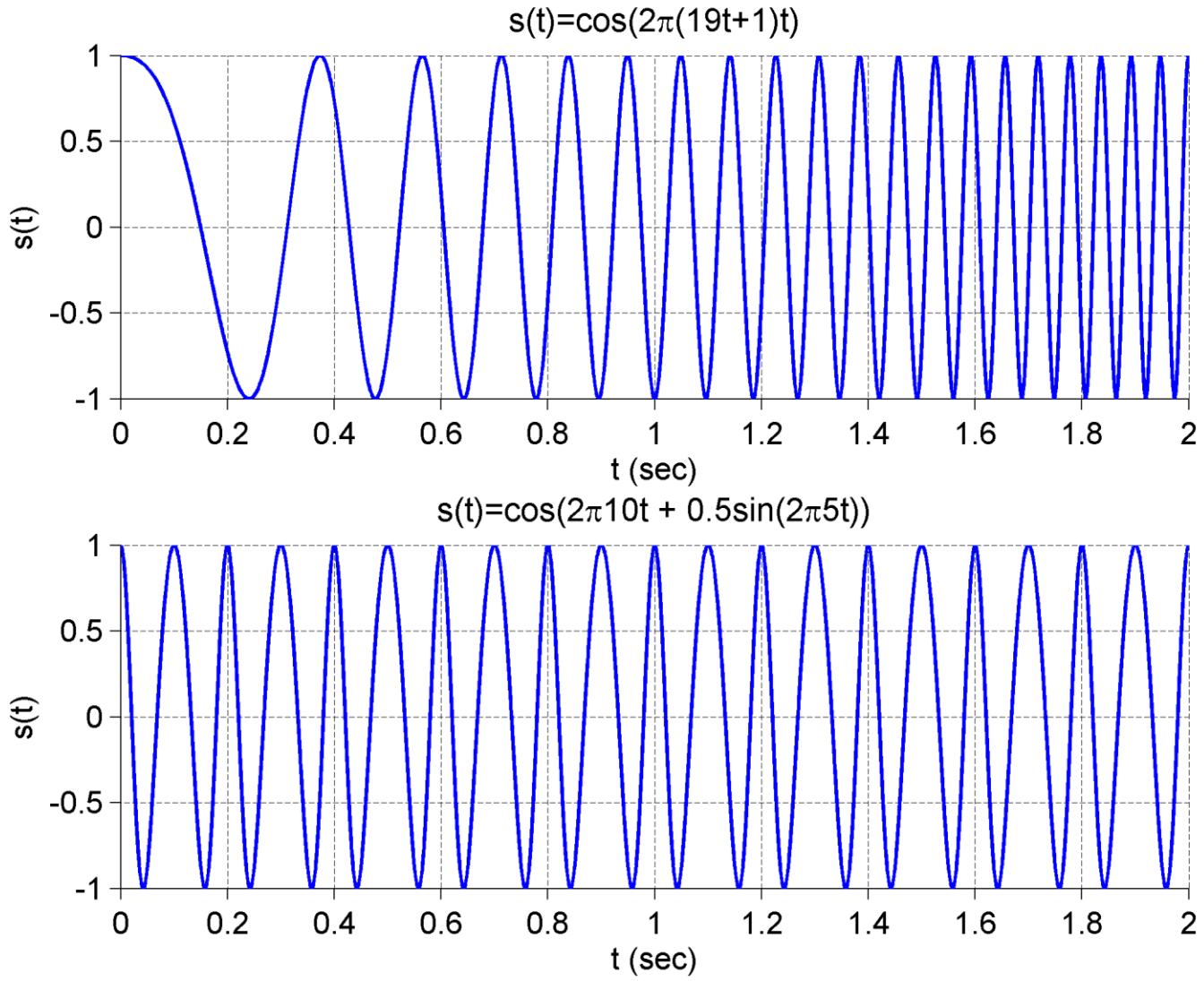
2.8 La fonction « max »



Exercice : Tracez le signal suivant



2.9 La composition de signaux : $s(t)= s_1 \circ s_2(t) = s_1(s_2(t))$

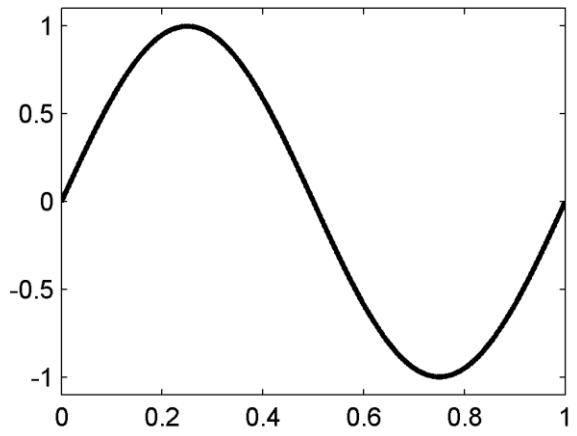


Chapitre 3. Les signaux numériques *(à temps discret)*

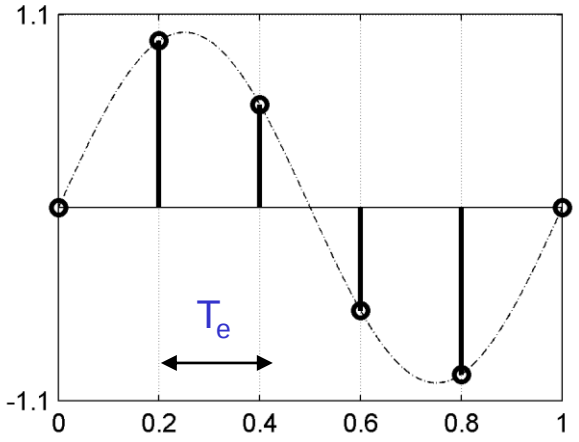
3.1: Passage du continu au numérique

amplitude

temps continu



temps discret

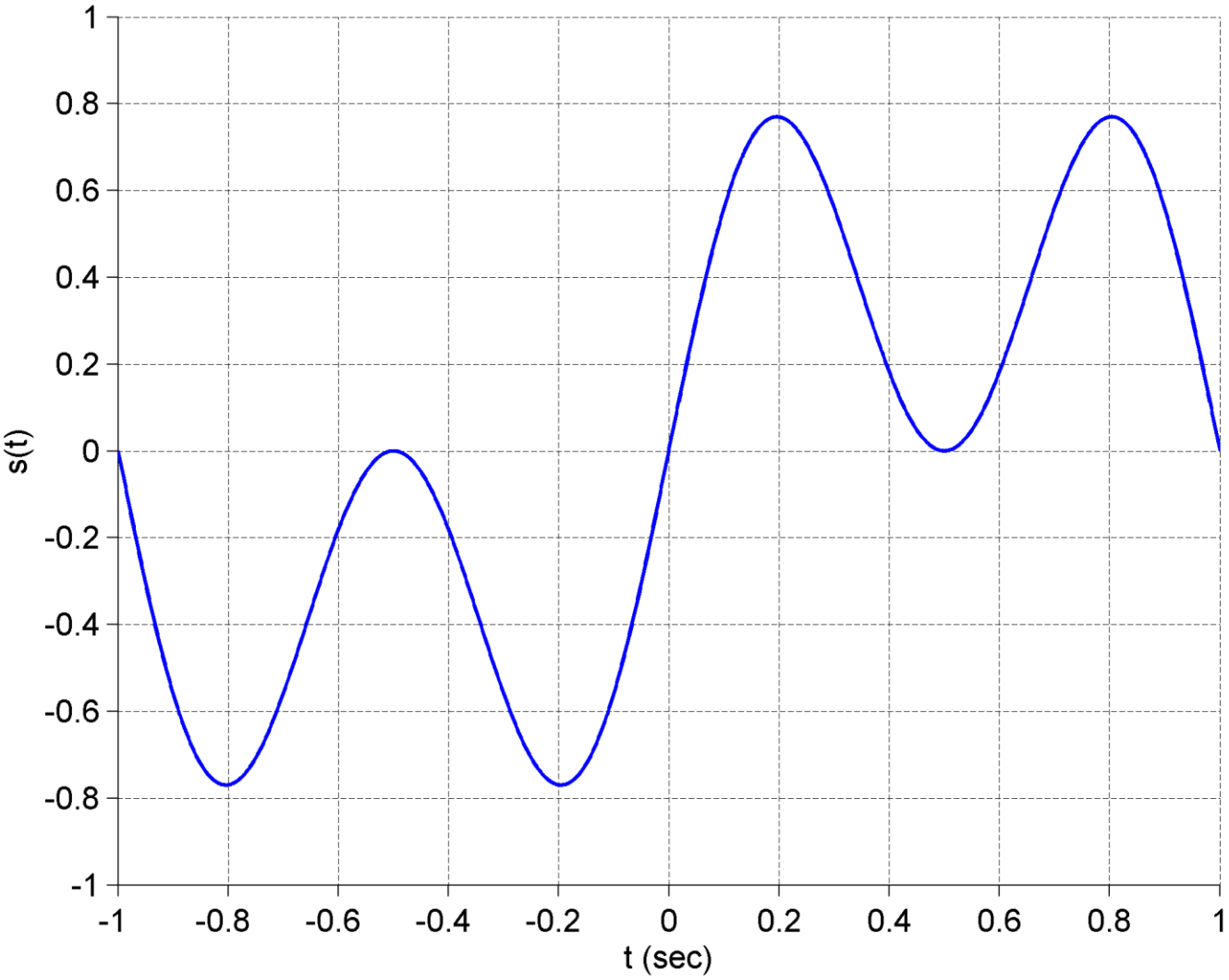


T_e: Cadence d 'échantillonnage (dépend de la fréquence d'échantillonnage)

$T_e = 1/ F_e$

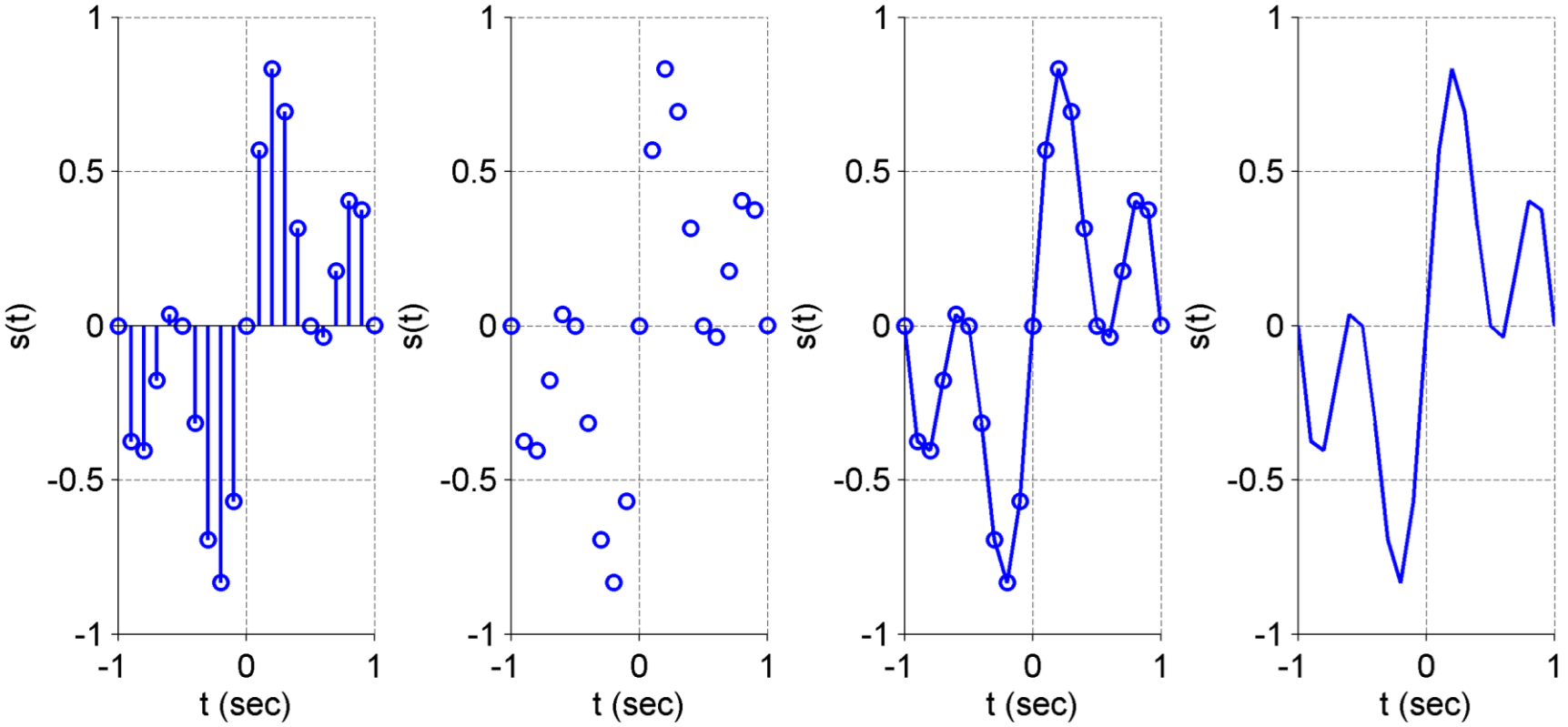
3.2: Exercice : Echantillonné le signal suivant à la fréquence d'échantillonnage F_e=10 Hz

t (sec)	s(t)
-1,000	0,000
-0,900	-0,559
-0,800	-0,769
-0,700	-0,559
-0,600	-0,182
-0,500	0,000
-0,400	-0,182
-0,300	-0,559
-0,200	-0,769
-0,100	-0,559
0,000	0,000
0,100	0,559
0,200	0,769
0,300	0,559
0,400	0,182
0,500	0,000
0,600	0,182
0,700	0,559
0,800	0,769
0,900	0,559
1,000	0,000



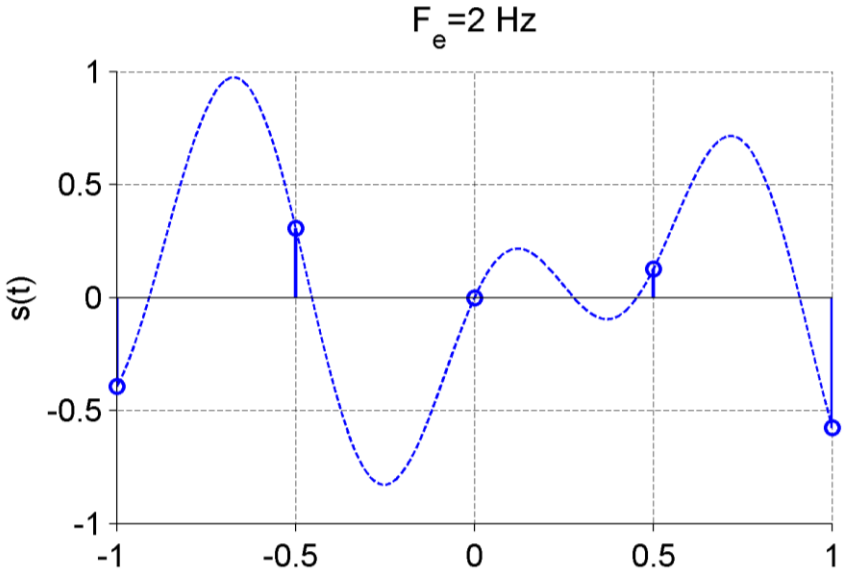
3.3: Représentations des signaux numériques

1. Echantillons et impulsions
2. Echantillons
3. Echantillons et interpolation
4. Interpolation

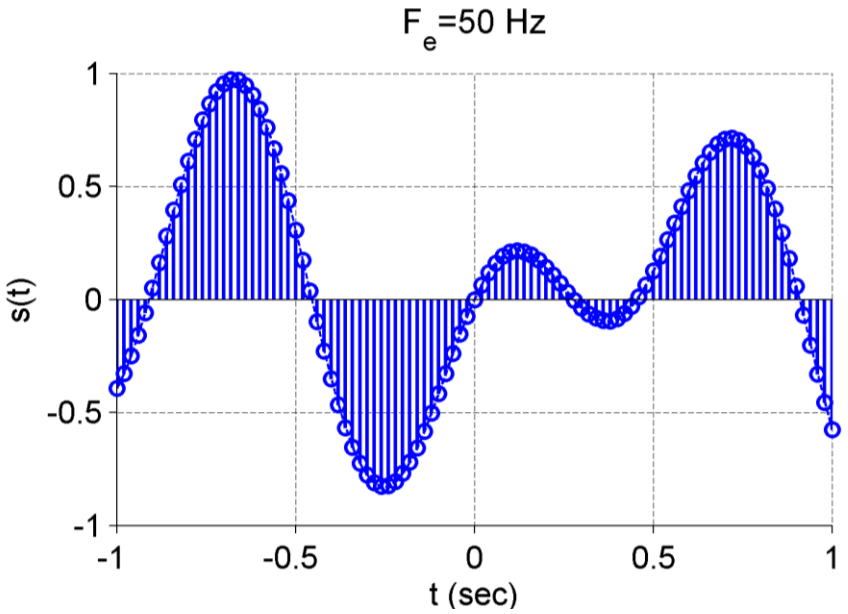


3.3: Les limites haute et basse de la numérisation

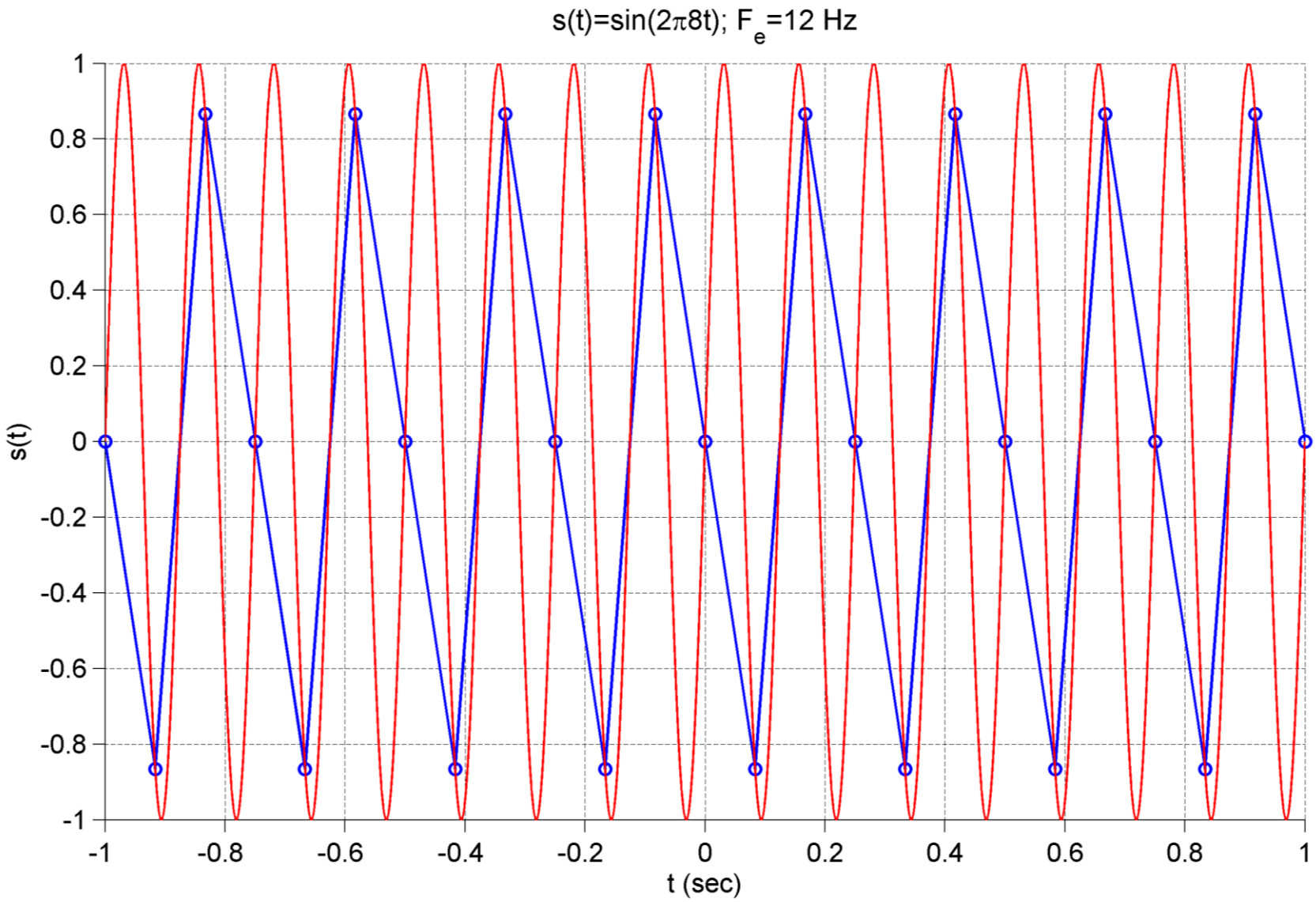
Fe trop petit:



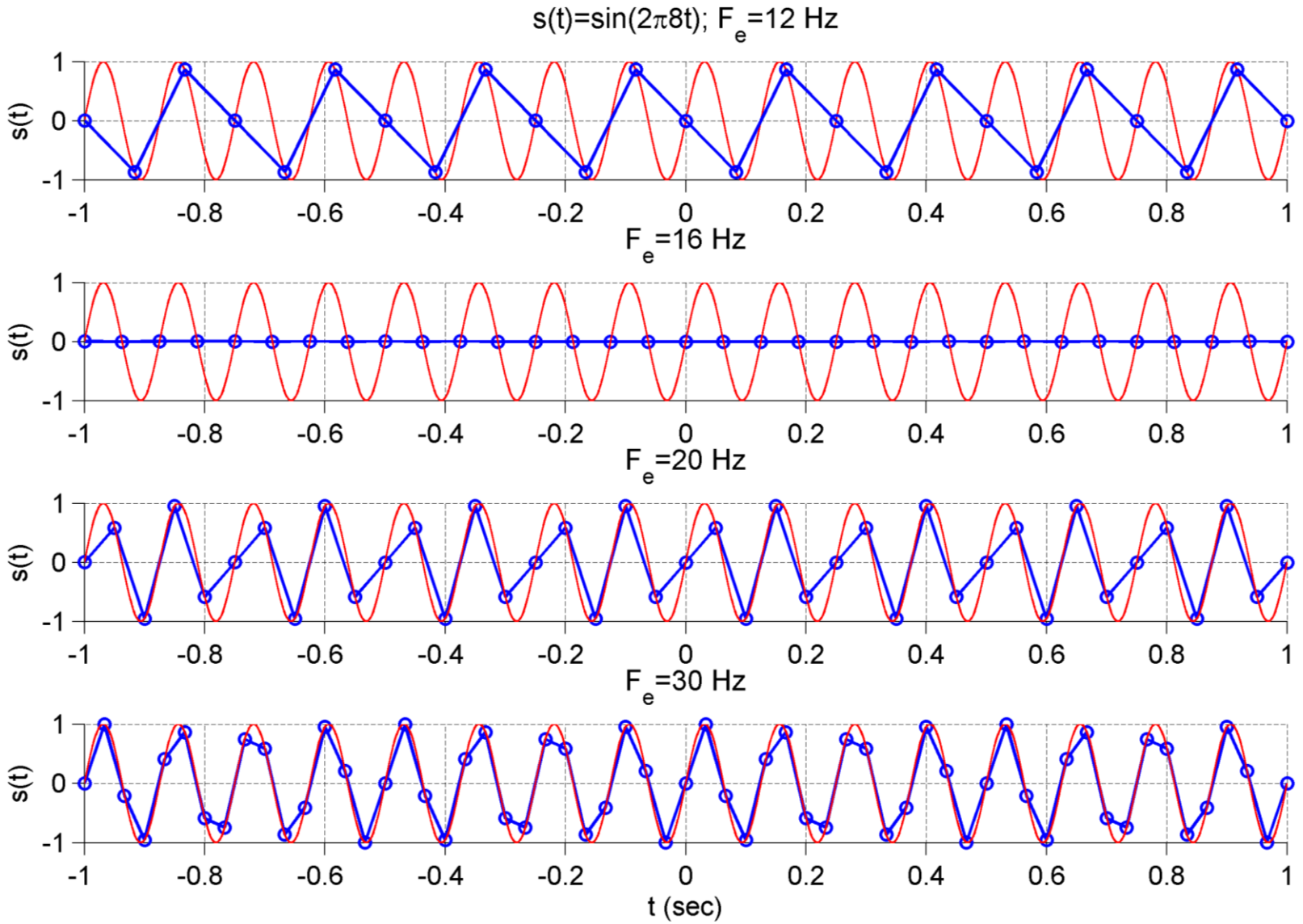
Fe trop grand:



3.3: Un exemple



3.3: Un exemple



3.4: Le théorème de l ’échantillonnage *(aussi appelé théorème de Shannon)*

Il est possible de numériser un signal analogique $s(t)$ à la cadence $T_e = 1/F_e$, de traiter le signal numérique, puis de reconstruire un signal analogique sans perte si:

- le signal $s(t)$ est à bande de fréquence B limitée

$$B = [\, f_m, f_M \,]$$

- la fréquence d ’échantillonnage vérifie

$$f_M < \frac{F_e}{2}$$

3.5: Exercice : Choisissez convenablement F_e

$s(t)=\sin(2\pi 1000t)$·

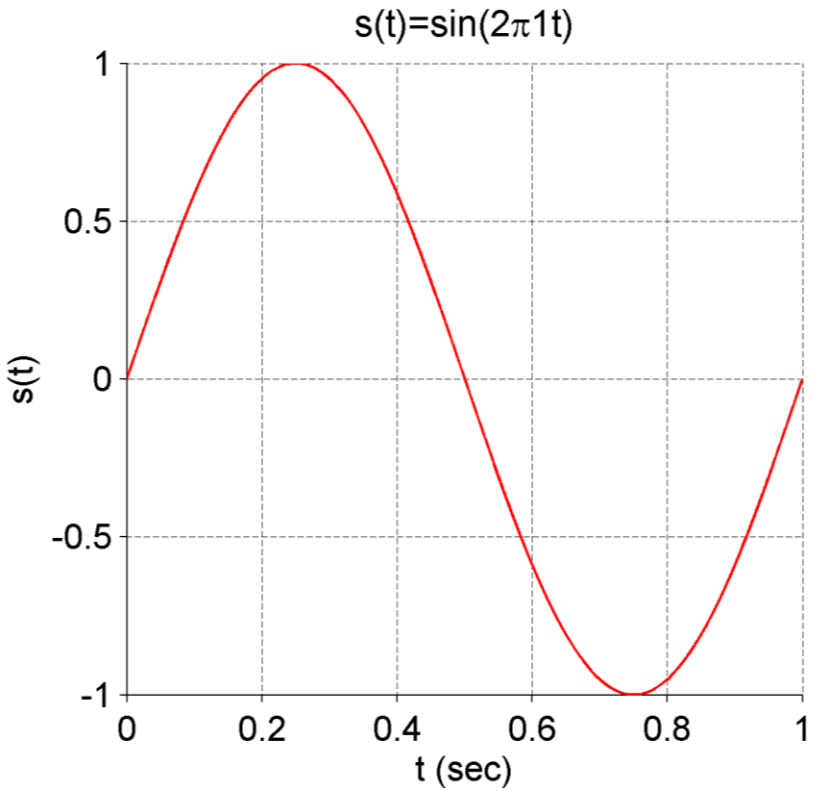
$s(t)=\sin(2\pi 1000t)+\sin(2\pi 2000t)$·

$s(t)=\sin(2\pi 1000t)+\sin(2\pi 2000t)+\sin(2\pi 3000t)$

Chapitre 4. Mesures sur les signaux

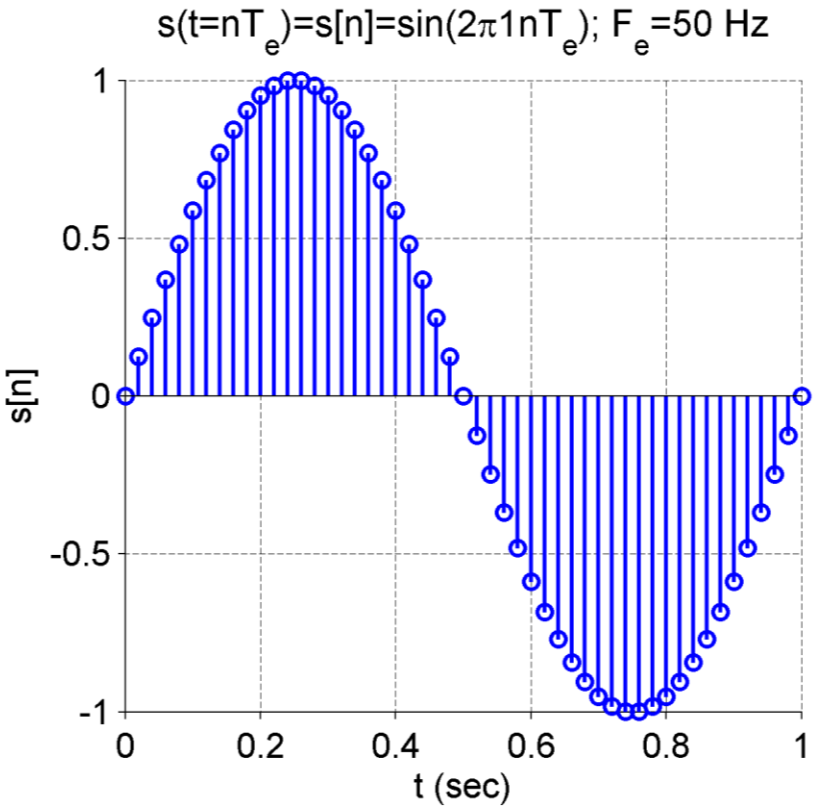
4.1. Valeur moyenne d'un signal périodique

En continu:



$$\bar{s} = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} s(t)\, dt$$

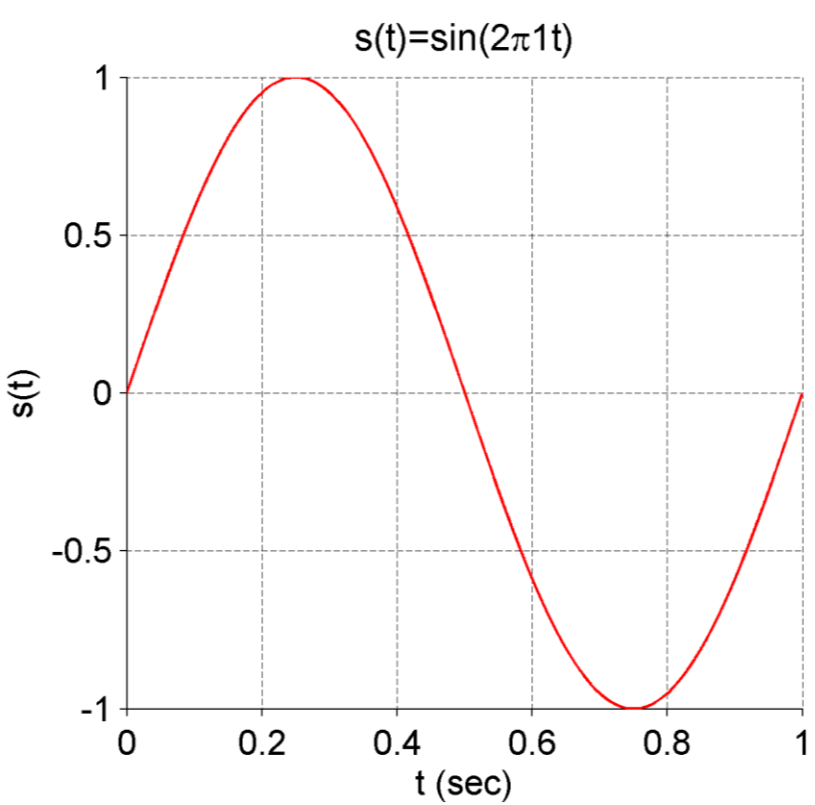
En numérique:



$$\bar{s} = \frac{1}{N} \sum_{n=0}^{N-1} s[n]$$

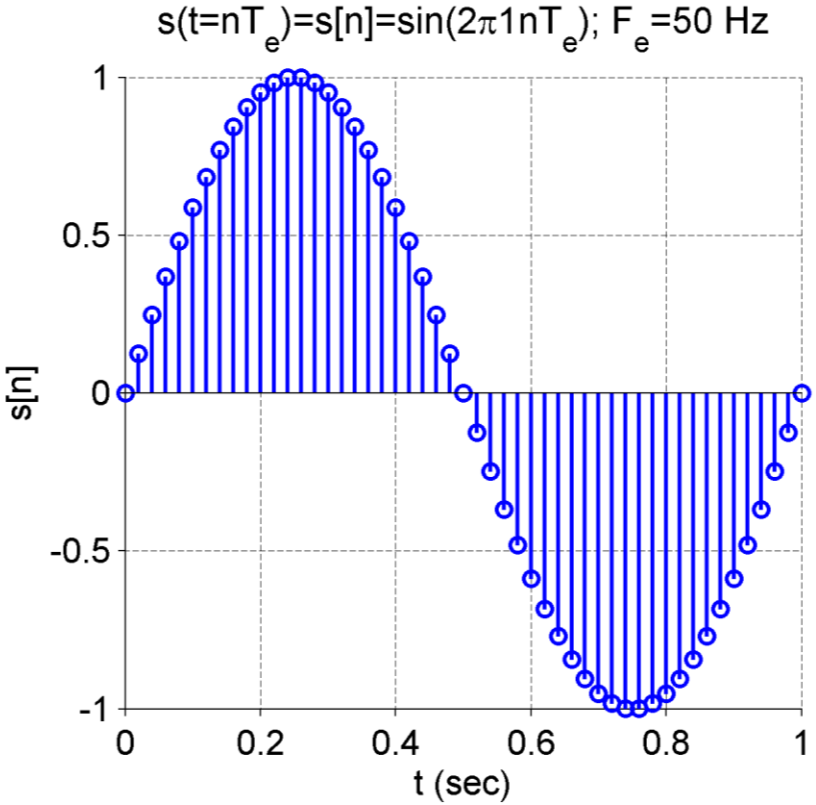
4.2. Puissance moyenne d'un signal périodique

En continu:



$$P_S = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} (s(t) - \bar{s})^2 \, dt$$

En numérique:



$$P_s = \frac{1}{N} \sum_{n=0}^{N-1} (s[n] - \bar{s})^2$$

4.3. Valeur efficace d'un signal périodique

En continu:

$$P_S = V_{eff}^2$$

$$\Leftrightarrow \quad V_{eff} = \sqrt{P_s}$$

En numérique:

$$P_S = V_{eff}^2$$

$$\Leftrightarrow \quad V_{eff} = \sqrt{P_s}$$

Correspondance **Traitement du signal / Statistiques**:

1. La **puissance** est au signal ce que la **variance** est aux statistiques
2. La **valeur efficace** est au signal ce que **l'écart-type** est aux statistiques

4.3. Exercice 1 : calculez la moyenne, la puissance moyenne et l'écart-type du signal

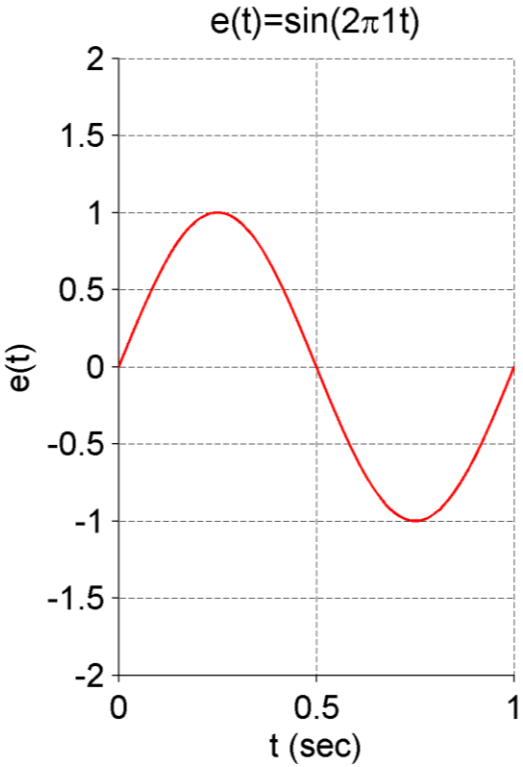
$$s_1(t) = A\sin(2\pi f_0 t)$$

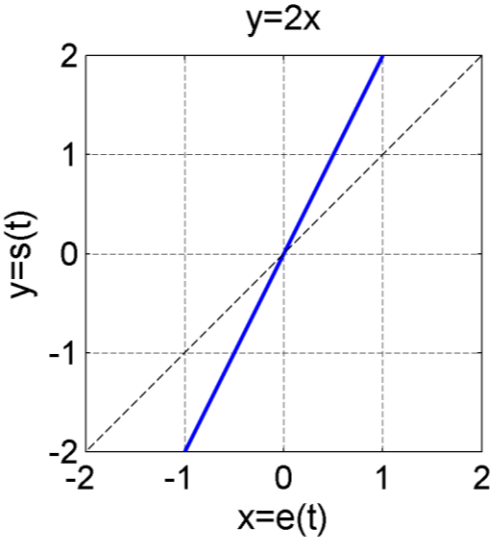
4.3. Exercice 2 : Calculez la moyenne, la puissance moyenne et l'écart-type d'un signal **carré** d'amplitude A et de fréquence f_0

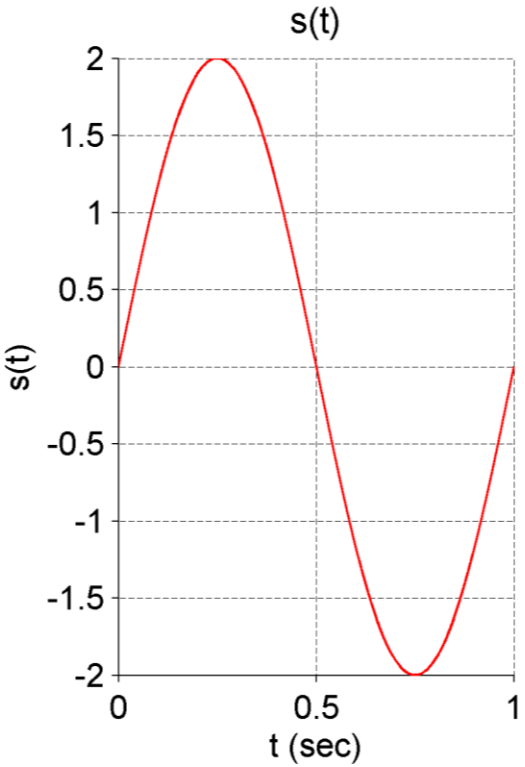
4.3. Exercice 3 : Calculez la moyenne, la puissance moyenne et l'écart-type d'un signal **rampe** d'amplitude A et de fréquence f_0

Chapitre 5. Amplifications linéaire et non-linéaire

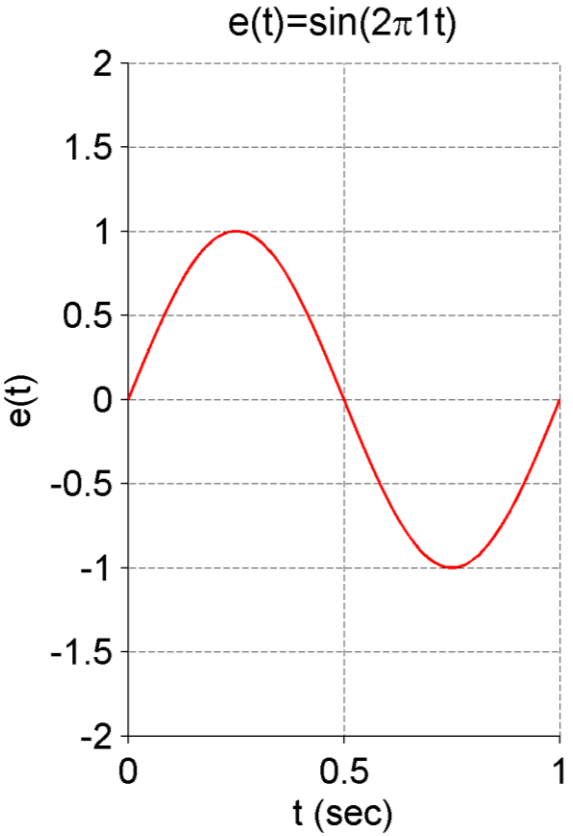
5.1. Amplification linéaire

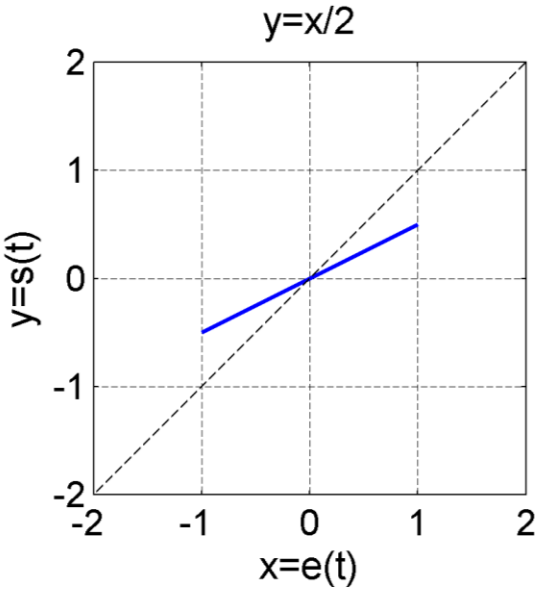


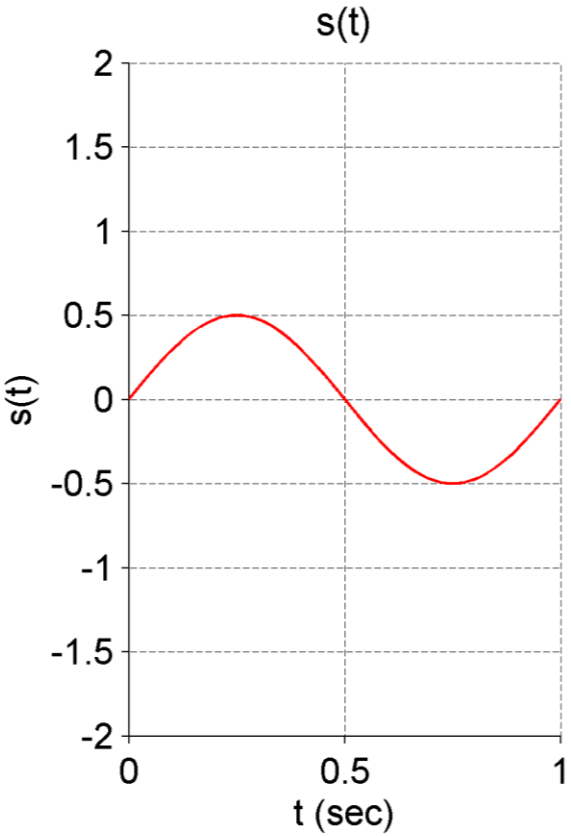




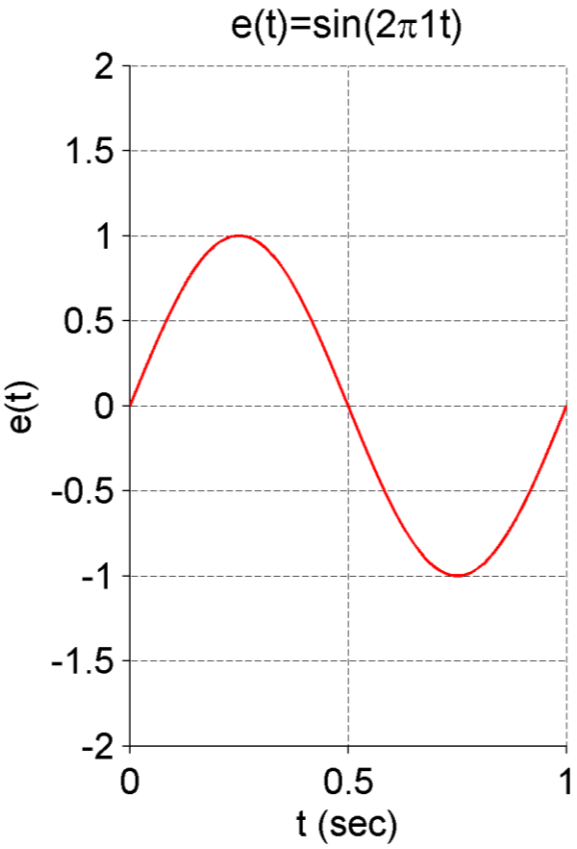
5.2. Compression linéaire

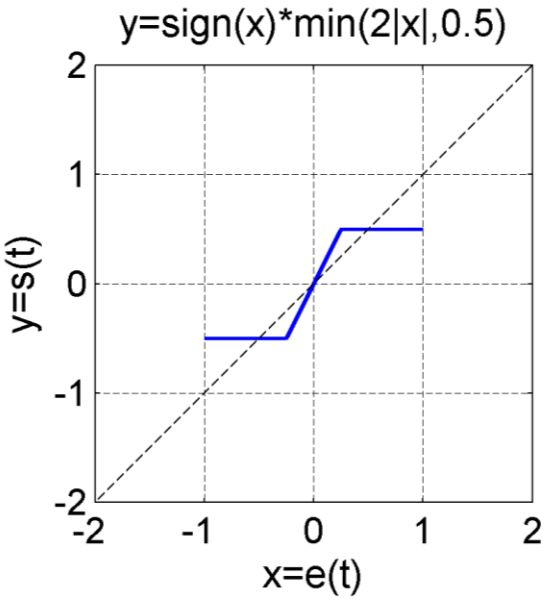


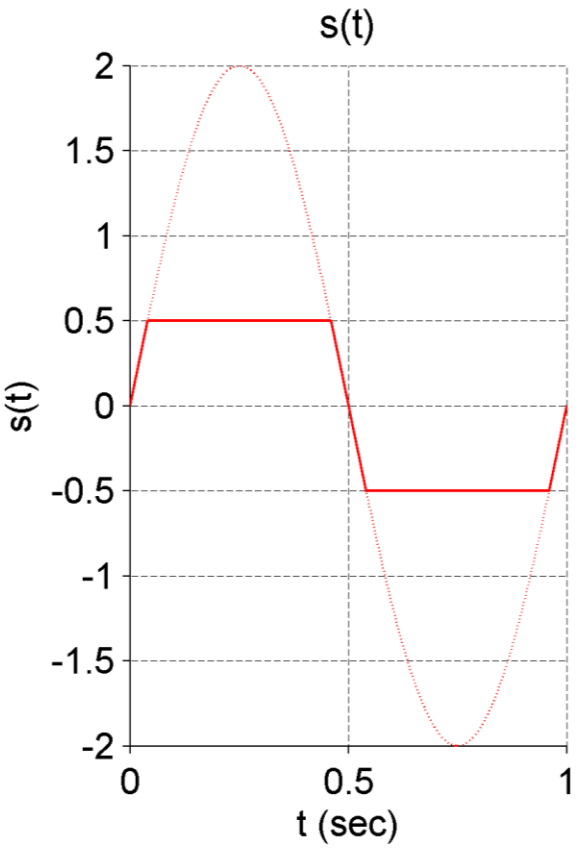




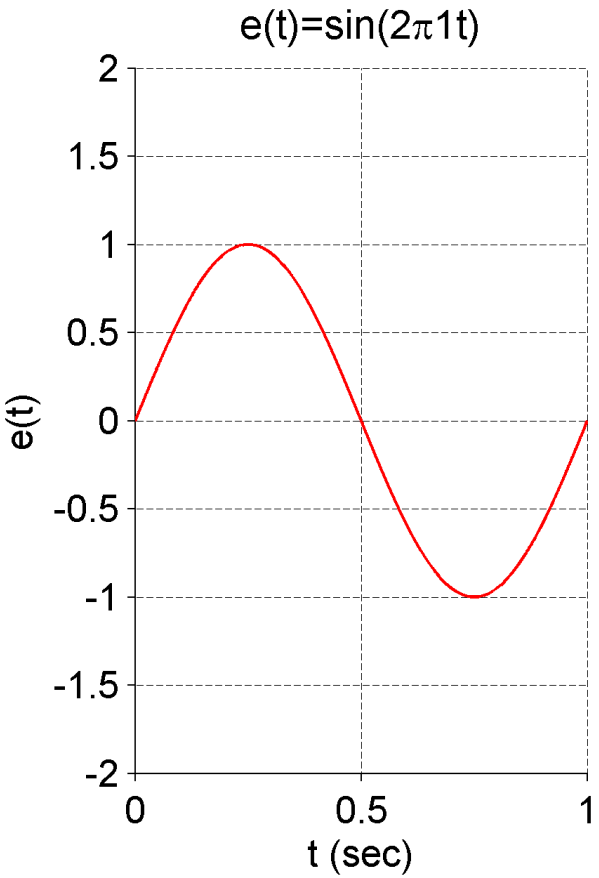
5.3. Amplification linéaire et écrêtage

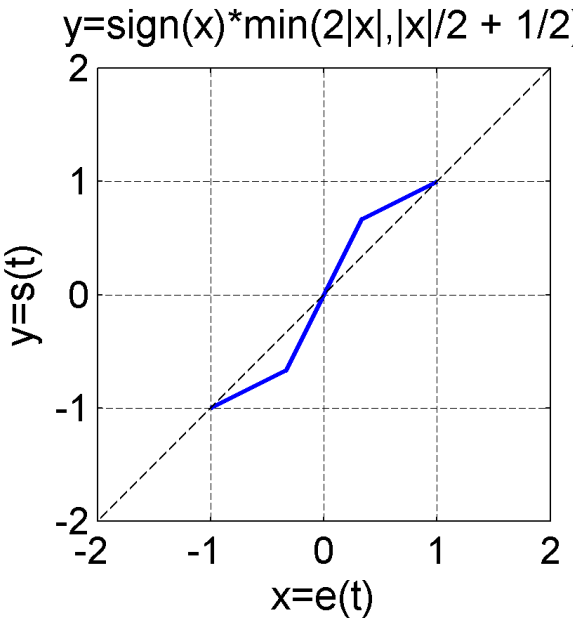


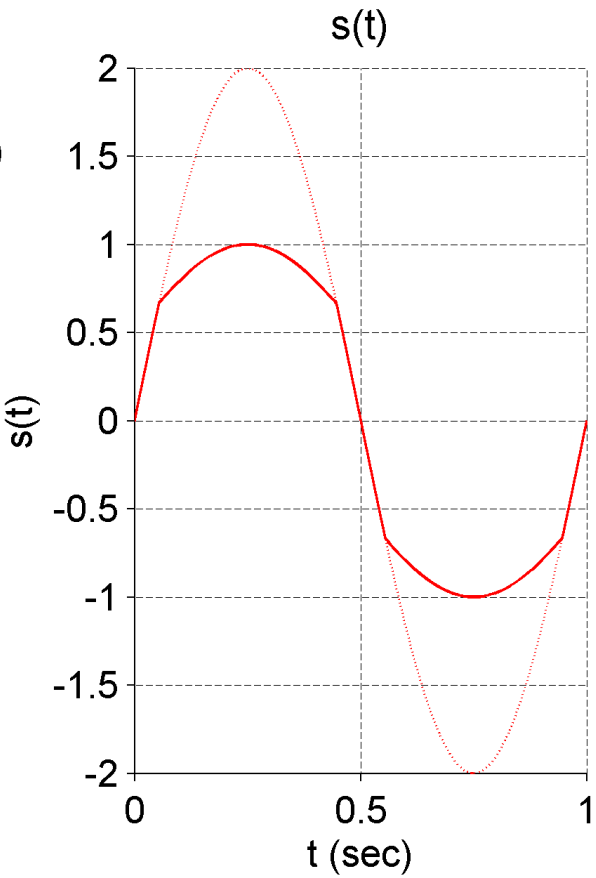




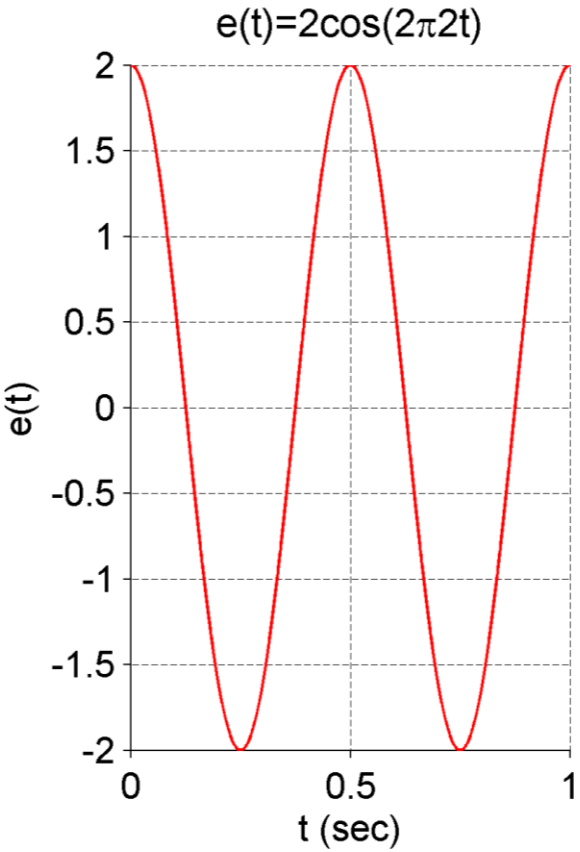
5.4. Amplification linéaire et compression linéaire

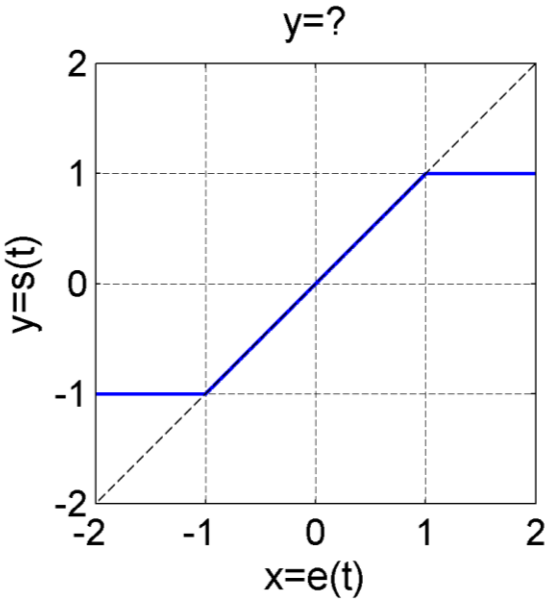


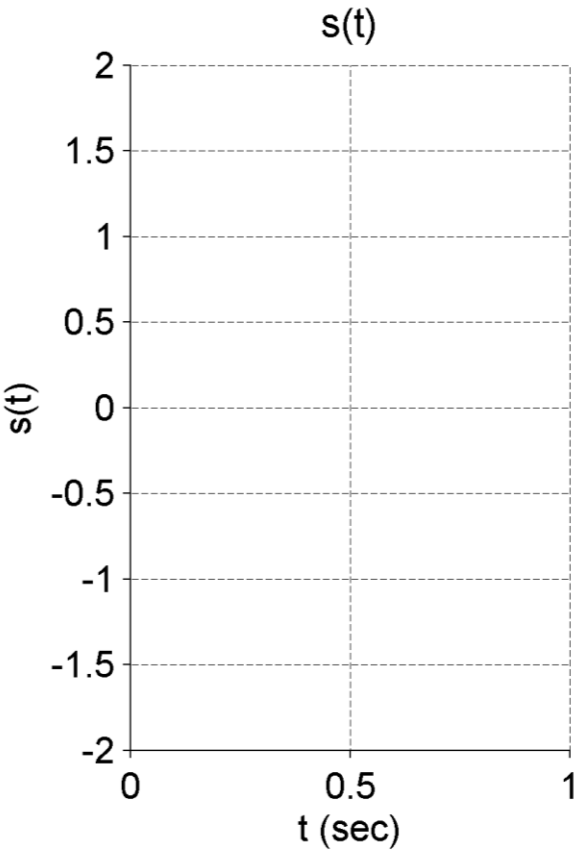




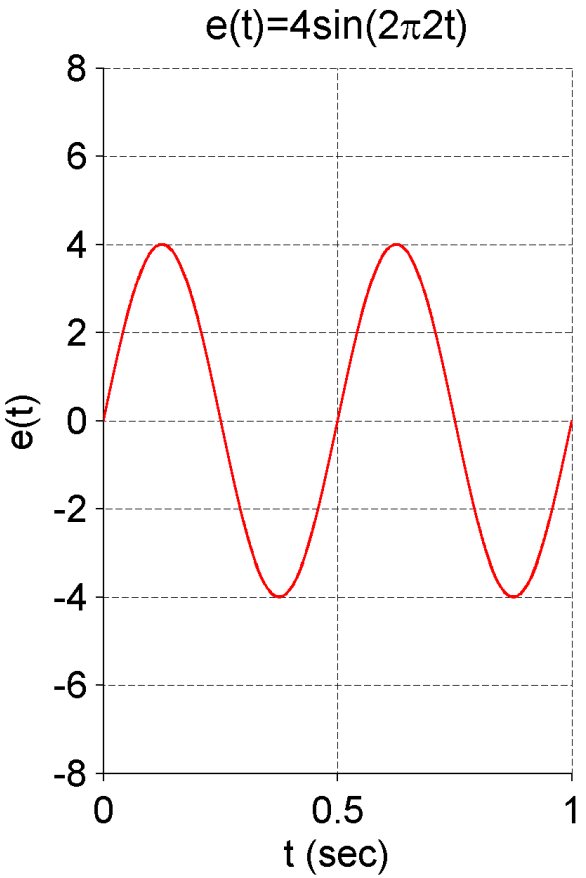
5.5. Exercice 1 : déterminez l'expression *y=f(x)* et tracez *s(t)*

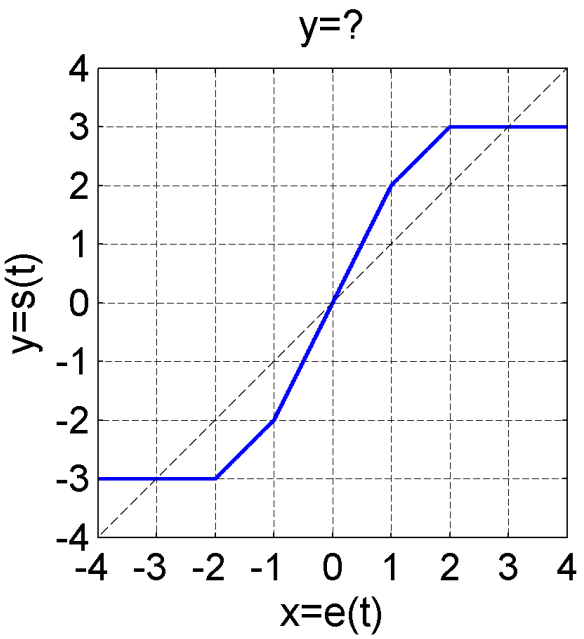


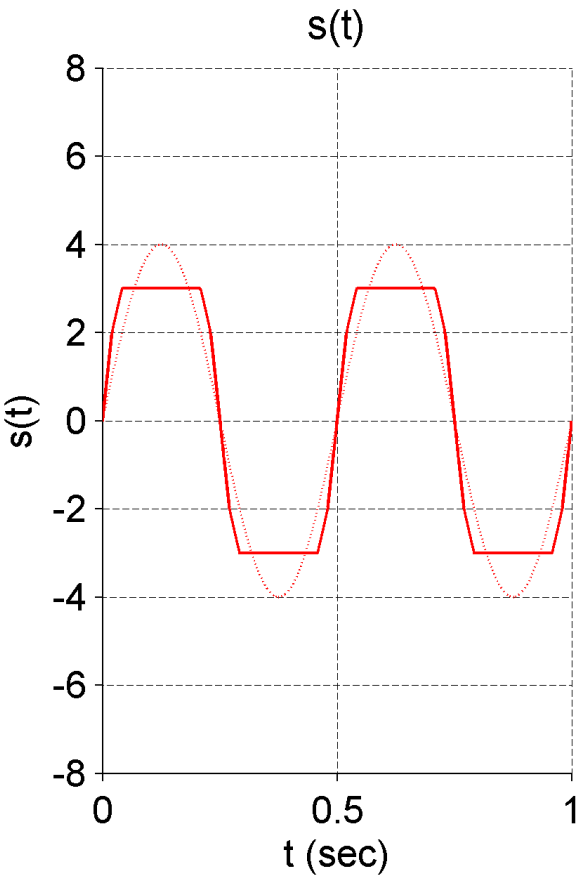




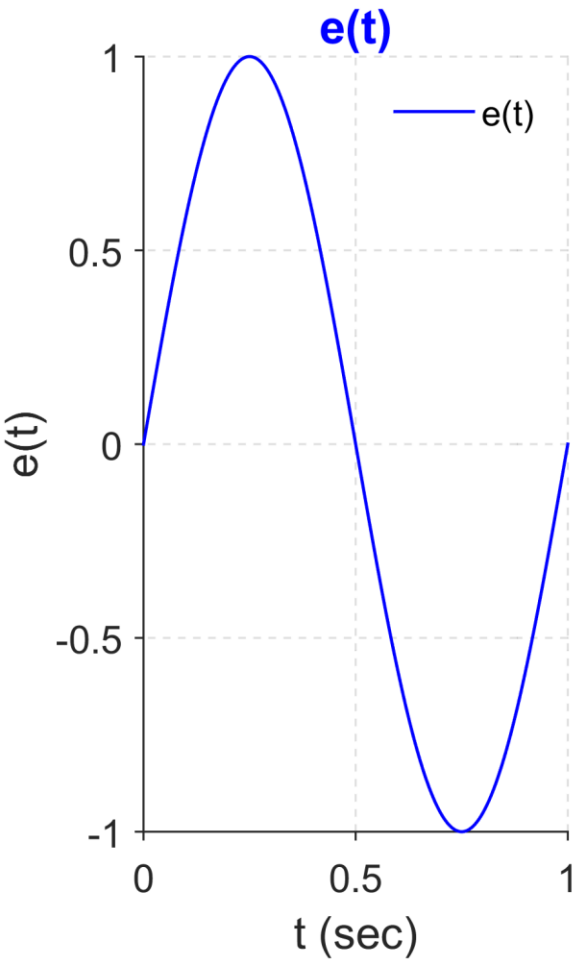
5.5. Exercice 2 : déterminez l'expression *y=f(x)*

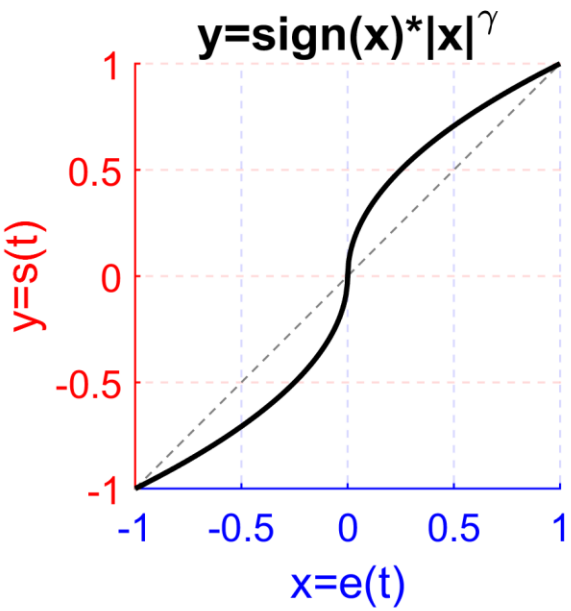


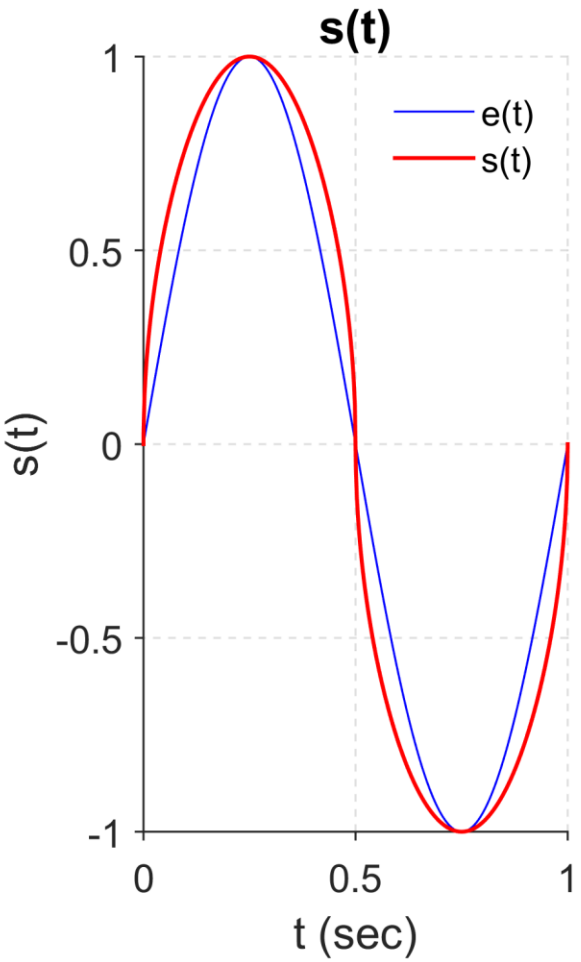




5.6. Le modèle γ







5.6. Le modèle γ avec une partie linéaire