Quantum Field Theory, problem sheet 14

Solutions to be discussed on 28/01/2025.

Problem 1: Furry's theorem

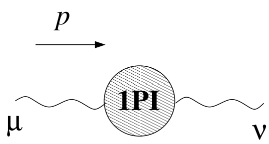
Charge conjugation is a linear operation $\mathcal{C}$ exchanging particles with antiparticles without involving any of their space-time transformation properties (such as spin or parity). We work in the Weyl basis, where charge conjugation acts on the Dirac field as $\psi \to C\overline{\psi}^T$ with $C = i\gamma^0\gamma^2$.

1. Show that $\gamma^2\gamma^\mu\gamma^2 = (\gamma^\mu)^*$.
2. Find the transformation of the electromagnetic current j_μ under $\mathcal{C}$.

   Hint: Remember that the components of the Dirac spinor ψ are Grassmann fields, and use the symmetry and reality properties of the γ matrices.
3. Find the how the photon A_μ must transform in order for the QED Lagrangian to be invariant under charge conjugation. Thus, show that all $(2n+1)$-photon amplitudes are zero. This is *Furry's theorem.*

Problem 2: Photon self-energy

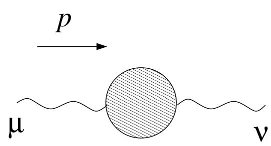
Show that, if $i\Pi^{\mu\nu}(p)$ is defined to be the sum of all 1-particle irreducible insertions into the photon propagator,

$i\Pi^{\mu\nu}(p) =$ 

then $\Pi^{\mu\nu}(p)$ can be written as

$$\Pi^{\mu\nu}(p) = (p^2 g^{\mu\nu} - p^\mu p^\nu)\,\Pi(p^2)$$

for a suitable function $\Pi(p^2)$. Thus, show that the full photon propagator takes the form



$$= \frac{-i}{p^2(1-\Pi(p^2))}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) - i\xi\frac{p^\mu p^\nu}{(p^2)^2}\,.$$

It follows that the photon mass remains zero to all orders in perturbation theory.