M2 Cosmos, Champs et Particules — Faculté des sciences de Montpellier

Quantum Field Theory, problem sheet 9

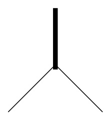
Problem 1: Two real scalar fields

Consider the following Lagrangian, involving two real scalar fields ϕ and χ which interact via a coupling μ:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}m^2\chi^2 - \frac{1}{2}M^2\phi^2 - \frac{1}{2}\mu\,\phi\chi^2.$$

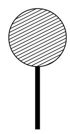
1. What is the mass dimension of μ? Is the theory renormalizable?

2. Derive the momentum-space Feynman rules for this theory, disregarding possible counterterms. In particular, show that there is a vertex attaching to two χ propagators (thin lines) and one ϕ propagator (thick line) for which the Feynman rule is



$$= -i\mu\,.$$

3. Draw the leading-order Feynman diagram contributing to the 1-point function of ϕ and determine its symmetry factor. In the following we will assume that there is a counterterm

 , such that  $= 0$.

(Here the blob stands for the sum of all contributions to the 1-point function.)

4. With this in mind, draw the Feynman diagrams contributing to the 2-point functions of ϕ and χ to order μ^2, and determine their symmetry factors. Are the higher-order corrections divergent? Evaluate the amputated diagrams in momentum space in dimensional regularization. You can use the results of the lecture without rederiving them.

5. Draw all leading-order Feynman diagrams contributing to $\chi\chi \to \chi\chi$ scattering, to $\phi\phi \to \phi\phi$ scattering and to $\phi\phi \to \chi\chi$ scattering.

6. Suppose that $M > 2m$, so that the decay $\phi \to \chi\chi$ is kinematically possible. Calculate matrix element $\mathcal{M}_{\phi\to\chi\chi}$ and the lifetime $\tau = 1/\Gamma$ of ϕ to leading order in μ. You can use the formula for the differential decay width in the rest frame (p_i is the initial-state four-momentum, p'_f are the final-state particle four-momenta and N is a combinatorial factor given by $n_f!$ for each group of n_f indistinguishable final-state particles)

$$\mathrm{d}\Gamma = \frac{1}{N}\frac{1}{2M}\prod_f \left(\widetilde{\mathrm{d}p'_f}\right)\, |\mathcal{M}|^2\,(2\pi)^4\,\delta^{(4)}\Big(p_i - \sum_f p'_f\Big)\,.$$