1) Les estimateurs des MCO sont des fonctions linéaires des Y_t de Y 1

$$\hat{\beta} = \frac{\sum x_t y_t}{\sum x_t^2} = \frac{\sum x_t (Y_t - \bar{Y})}{\sum x_t^2} = \frac{\sum x_t Y_t}{\sum x_t^2} - \frac{\sum x_t \bar{Y}}{\sum x_t^2}$$

$$\sum x_t = \sum (X_t - \bar{X}) = n\bar{X} - n\bar{X} = 0$$

$$\hookrightarrow \boxed{\hat{\beta}} = \frac{\sum x_t Y_t}{\sum x_t^2} = \boxed{\sum w_t Y_t}$$

avec $w_t = \dfrac{x_t}{\sum x_t^2}$

Propriétés des w_t

- $\sum w_t = \dfrac{\sum x_t}{\sum x_t^2} = 0$
- $\sum w_t^2 = \sum \left(\dfrac{x_t}{\sum x_t^2} \right)^2 = \dfrac{1}{\sum x_t^2}$
- $\sum x_t w_t = \sum w_t (X_t - \bar{X}) = \sum X_t w_t = 1$

car $\sum x_t w_t = \dfrac{\sum x_t \cdot x_t}{\sum x_t^2} = 1$

$$\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X} = \frac{1}{n}\sum Y_t - \left(\sum w_t Y_t\right)\bar{X}$$

$$\boxed{\hat{\alpha} = \sum \left(\frac{1}{n} - \bar{X} w_t\right) Y_t}$$

D'où la propriété!

4) $\hat{\alpha}$ et $\hat{\beta}$ sont des estimateurs convergents

- $V[\hat{\beta}] = \frac{\sigma_\varepsilon^2}{\sum x_t^2} = \frac{\sigma_\varepsilon^2}{\sum (X_t - \bar{X})^2} = \frac{\sigma_\varepsilon^2}{n S^2} \xrightarrow[n \to +\infty]{} 0$

- $V[\hat{\alpha}] = \sigma_\varepsilon^2 \frac{\sum x_t^2}{n \sum x_t^2} \xrightarrow[n \to +\infty]{} 0$