M2 Cosmos, Champs et Particules — Faculté des sciences de Montpellier

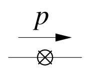
# Quantum Field Theory, problem sheet 8

## Problem 1: Counterterms in $\phi^4$ theory

The Lagrangian of $\phi^4$ theory in renormalised perturbation theory is

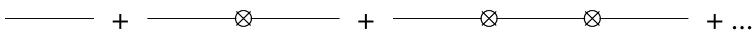
$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_r\partial^\mu\phi_r - \frac{1}{2}m^2\phi_r^2 - \frac{\lambda}{24}\phi_r^4 + \frac{1}{2}\delta_Z\,\partial_\mu\phi_r\partial^\mu\phi_r - \frac{1}{2}\delta_{m^2}\phi_r^2 - \frac{\delta_\lambda}{24}\phi_r^4\,.$$

Here $\phi_r$ is the rescaled field $\phi_r = \phi/\sqrt{Z}$, and $\delta_Z \equiv Z - 1$, $\delta_{m^2}$ and $\delta_\lambda$ are the counterterms.
Show that the Feynman rule for the "2-point vertex" associated to the counterterms $\delta_Z$ and $\delta_{m^2}$ is




$$= i(p^2\delta_Z - \delta_{m^2})\,.$$

*Hint:* Note that the two-point function in the limit $\lambda \to 0$, $\delta_\lambda \to 0$ is now given by the infinite series




By summing this series, show that it corresponds to the Feynman propagator for a a rescaled field $\sqrt{Z}\phi_r$ with a shifted mass $m^2 + \delta_{m^2}$.

## Problem 2: Feynman parameters

Prove the following identities ($A \neq 0$ and $B \neq 0$ are real constants):

1. $$\frac{1}{AB} = \int_0^1 \mathrm{d}x\,\frac{1}{(xA + (1-x)B)^2}$$

2. $$\frac{1}{A^n B} = \int_0^1 \mathrm{d}x\,\frac{nx^{n-1}}{(xA + (1-x)B)^{n+1}}$$

3. $$\frac{1}{A_1\cdots A_n} = \int_0^1 \mathrm{d}x_1\cdots\mathrm{d}x_n\,\delta(1 - x_1 - \ldots - x_n)\,\frac{(n-1)!}{(x_1A_1 + \ldots + x_nA_n)^n}$$