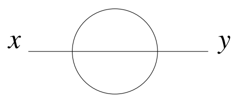
Quantum Field Theory, problem sheet 7

Solutions to be discussed on 19/11/2024.

Problem 1: Feynman diagrams

For ϕ^4 theory:

1. Find all connected Feynman diagrams contributing to the four-point function at $\mathcal{O}(\lambda^2)$ and determine their symmetry factors.
2. Repeat this exercice for the six-point function at $\mathcal{O}(\lambda^2)$.
3. State the algebraic expression in momentum space which corresponds to the following Feynman diagram (without evaluating the integrals)



An even simpler quantum field theory is ϕ^3 theory: $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\mu}{6}\phi^3$. It is problematic because its Hamiltonian is not bounded from below, but this is not visible in perturbation theory, so one may still formally construct its n-point functions perturbatively.

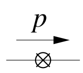
4. Find the Feynman rules for ϕ^3 theory, draw the connected Feynman diagrams contributing to the 1-point, 2-point and 3-point functions at $\mathcal{O}(\mu^2)$, and determine their symmetry factors.

Problem 2: Counterterms in ϕ^4 theory

The Lagrangian of ϕ^4 theory in renormalised perturbation theory is

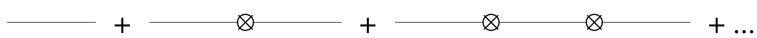
$$\begin{aligned}\mathcal{L} = {} & \frac{1}{2}\partial_\mu\phi_r\partial^\mu\phi_r - \frac{1}{2}m^2\phi_r^2 - \frac{\lambda}{24}\phi_r^4 \\ & + \frac{1}{2}\delta_Z\,\partial_\mu\phi_r\partial^\mu\phi_r - \frac{1}{2}\delta_{m^2}\phi_r^2 - \frac{\delta_\lambda}{24}\phi_r^4\,.\end{aligned}$$

Here ϕ_r is the rescaled field $\phi_r = \phi/\sqrt{Z}$, and $\delta_Z \equiv Z - 1$, δ_{m^2} and δ_λ are the counterterms.
Show that the Feynman rule for the "2-point vertex" associated to the counterterms δ_Z and δ_{m^2} is



$$= i(p^2\delta_Z - \delta_{m^2})\,.$$

Hint: Note that the two-point function in the limit $\lambda \to 0$, $\delta_\lambda \to 0$ is now given by the infinite series



By summing this series, show that it corresponds to the Feynman propagator for a a rescaled field $\sqrt{Z}\phi_r$ with a shifted mass $m^2 + \delta_{m^2}$.