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Prior distributions

Exercise 1

We consider an n-sample $(x_1, \ldots, x_n)$ following a Rayleigh distribution with parameter $\sigma^2 > 0$ whose density is such that

$$f(x; \sigma^2) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right) \mathbb{I}_{x \geq 0} \, .$$

1 Give the Jeffreys prior distribution for the parameter σ^2. Is it a proper prior distribution ?

2 For Jeffreys prior distribution, give the Bayesian estimator $\hat{\sigma}_1^2$ associated with the quadratic loss function.

3 What is the family of conjugate prior for the parameter σ^2 ?

4 For the family of conjugate prior, give the Bayesian estimator $\hat{\sigma}_2^2$ associated with the quadratic loss function.

Exercise 2

The coach of a soccer team wants to rank his players according to their performance in penalty shoot-outs. He therefore seeks to estimate the intrinsic probability of each player scoring a penalty. To do this, over a 30-day period, he records the number of consecutive shots required to score a penalty against different goalkeepers.
We focus on the case of one player and assume that all trials are independent. We then have 30 independent realizations $x_1, \ldots, x_{30}$ following a geometric distribution with parameter $\theta \in [0, 1]$ where θ is the intrinsic probability that the player will score a penalty kick. We recall that, for any $i \in \{1, \ldots, 30\}$,

$$\mathbb{P}(X_i = x_i) = (1 - \theta)^{x_i - 1} \theta \times \mathbb{I}_{x_i \in \mathbb{N}_*} \, .$$

We recall that the expectation of a geometric distribution with parameter θ is equal to $1/\theta$. We place ourselves in the Bayesian paradigm.

1 Give the Jeffreys prior $\pi^J(\theta)$ for the parameter θ.

2 Give the posterior distribution of θ associated with $\pi^J(\theta)$.