

## QUANTUM FIELD THEORY, PROBLEM SHEET 13

Solutions to be discussed on 24/01/2024.

### Problem 1: Furry's theorem

*Charge conjugation* is a linear operation  $\mathcal{C}$  exchanging particles with antiparticles without involving any of their space-time transformation properties (such as spin or parity). We work in the Weyl basis, where charge conjugation acts on the Dirac field as  $\psi \rightarrow C\bar{\psi}^T$  with  $C = i\gamma^0\gamma^2$ .

1. Show that  $\gamma^2\gamma^\mu\gamma^2 = (\gamma^\mu)^*$ .
2. Find the transformation of the electromagnetic current  $j_\mu$  under  $\mathcal{C}$ .  
*Hint:* Remember that the components of the Dirac spinor  $\psi$  are Grassmann fields, and use the symmetry and reality properties of the  $\gamma$  matrices.
3. Find the how the photon  $A_\mu$  must transform in order for the QED Lagrangian to be invariant under charge conjugation. Thus, show that all  $(2n + 1)$ -photon amplitudes are zero. This is *Furry's theorem*.

### Problem 2: Photon self-energy

Show that, if  $i\Pi^{\mu\nu}(p)$  is defined to be the sum of all 1-particle irreducible insertions into the photon propagator,

$$i\Pi^{\mu\nu}(p) = \begin{array}{c} \xrightarrow{p} \\ \mu \text{ --- } \textcircled{\text{1PI}} \text{ --- } \nu \end{array}$$

then  $\Pi^{\mu\nu}(p)$  can be written as

$$\Pi^{\mu\nu}(p) = (p^2 g^{\mu\nu} - p^\mu p^\nu) \Pi(p^2)$$

for a suitable function  $\Pi(p^2)$ . Thus, show that the full photon propagator takes the form

$$\begin{array}{c} \xrightarrow{p} \\ \mu \text{ --- } \textcircled{\hspace{1cm}} \text{ --- } \nu \end{array} = \frac{-i}{p^2(1 - \Pi(p^2))} \left( g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) - i\xi \frac{p^\mu p^\nu}{(p^2)^2}.$$

It follows that the photon mass remains zero to all orders in perturbation theory.