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The Doppler effect is the same for both optics and acoustics

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ABSTRACT

According to modern physics, there are three main differences between the Doppler effect in optics and the Doppler effect in acoustics: (i) difference in which relative velocity is more important; (ii) difference in the transverse Doppler effect; (iii) difference in the frequency observed by a stationary observer when the source is moving vs. the frequency observed by a moving observer when the source is stationary. All three of these differences are examined in this paper and, in the end, it will be shown that the Doppler effect is the same for both - optics and acoustics.

1. Introduction

The fact that the Doppler effect (DE) is applied in many fields shows how important it is to physics and beyond. However, this effect is more important than it seems. This is because some of the basic meanings of the DE are not fully understood and some other meanings are misinterpreted. This paper will deal with the comparison of the DE in optics and acoustics. According to modern physics there are three main differences between the DE in optics and the DE in acoustics. The first difference is in which relative velocity is more important in the DE. According to modern physics, light doesn't require a propagation medium, whereas sound waves need such a medium and this fact leads to two different explanations of the DE for these two cases.

The second difference is in the transverse DE (TDE). The special theory of relativity (STR) claims that the TDE can only be explained by reference to this theory. According to this theory the TDE appears only for electromagnetic waves, thus not for sound waves.

The third difference is in the frequency observed by a stationary observer when the source is moving with velocity v toward the observer vs. the frequency observed by an observer moving with velocity v toward the stationary source. Again, the STR claims that in optics these two observed frequencies are the same and there is only one formula for both cases, while the "classical DE" in acoustics finds two different equations for these cases.

Do these three differences have a scientific basis? The answer is in what follows.

2. The "difference" in which relative velocity is more important in the DE

In physics textbooks [1–4] we find that the DE formula for electromagnetic waves is different from the DE formula for sound waves. In these textbooks we read that: "A Doppler effect also occurs for electromagnetic waves, but it differs from the Doppler effect for sound waves in two ways. First, in the Doppler effect for sound waves, motion relative to the medium is most important because sound waves require a medium in which to propagate. In contrast, the medium of propagation plays no role in the Doppler effect for electromagnetic waves because the waves require no medium in which to propagate. Second, the speed of sound that appears in the equation for the Doppler effect for sound depends on the reference frame in which it is measured. In contrast, the speed of electromagnetic waves has

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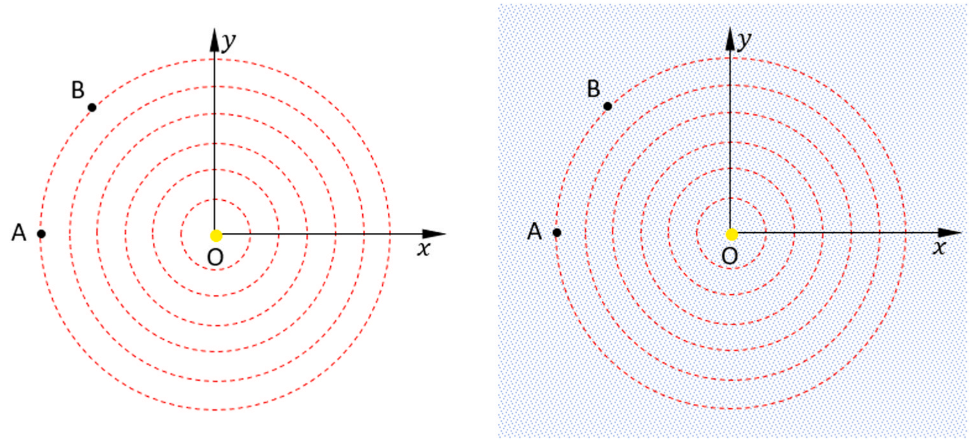

Fig. 1. Propagation of light relative to frame of reference with origin at O (a); propagation of sound relative to frame of reference with origin at O, which is connected to stationary air (*b*).

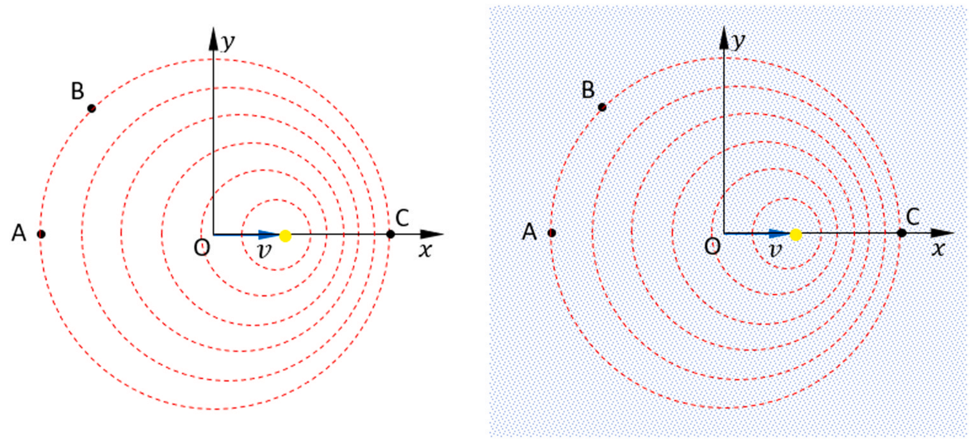

Fig. 2. The DE for light (a); the DE for sound (*b*).

the same value in all coordinate systems that are either at rest or moving at constant velocity with respect to one another" [2]. Thus, sound requires a medium in which to propagate, while light doesn't (of course, this applies if we are dealing with the propagation of light in the ideal vacuum. Otherwise, we must keep in mind the fact that any medium plays an active role in the speed of propagation of a wave, whatever kind it is [5,6]). Let us see if this distinction is enough to derive different DE formulas for these two cases.

Fig. 1*a* shows a light source in vacuum which is resting relative to the frame of reference *xOy*. In this frame all wavefronts have the same centre – the source at O. Fig. 1*b* shows a source of sound, which is resting relative to the air. In the air all wavefronts have the same centre – the source at origin O of reference frame *xOy*.

What is the difference between these two situations regarding the propagation of waves, except that one propagates without a medium and the other through a medium? There is no other difference. Each source emits a certain frequency and the same frequency is recorded by any resting observer, so there is no DE in either of the cases. Here we can see that the air is connected to a frame of reference with origin at O (at the source of the sound), and now we can say that the air and this frame of reference with origin O (Fig. 1b) represent the same frame of reference, because they are resting relative to each other. The sound waves propagate relative to the air (and relative to the frame of reference), since the waves move, whereas air as a whole is resting. We must always remember that.

Now let these two sources move relative to their respective frames. The DE will appear in both cases. What is the difference between these two situations regarding the DE? Again, there is no difference. In both cases the waves will propagate relative to their respective resting frame of reference – the light waves move relative to the frame of reference with its origin at O (Fig. 2*a*) and the sound waves move relative to the air and, at the same time, also relative to the frame of reference with its origin at O (Fig. 2*b*). Therefore, there is no reason that we would obtain different DE formulas for these cases caused by the absence/ presence of a stationary medium. For the DE it is not important what carries the wave, just as it is not important whether the wave needs the medium or not. It is known that when a wave is propagated in a medium, this wave does not shift the medium, so the medium which is connected to a resting system (the reference frame with origin at O, Fig. 2*b*) does not play a role in the DE formula. The DE deals with the propagation of the wave, but not with the method of its creation.

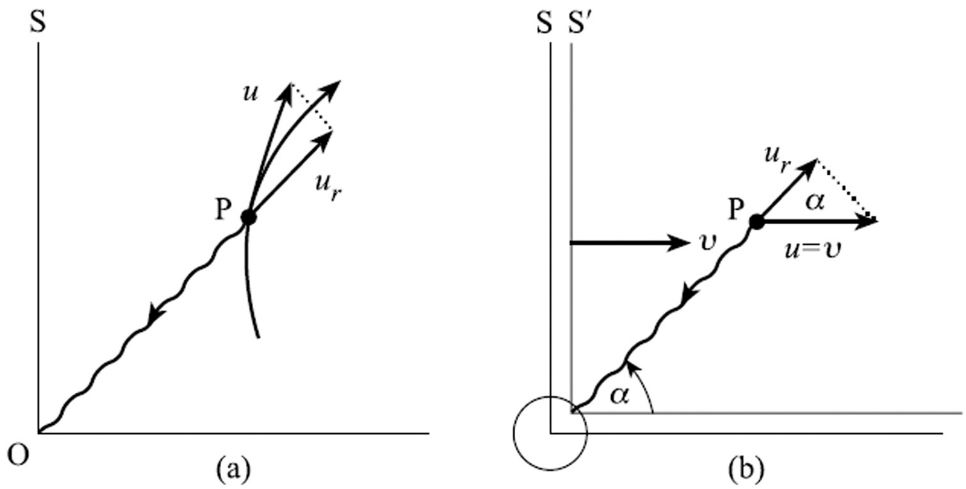

Fig. 3. W. Rindler's light source moves relative to reference frame S [4].

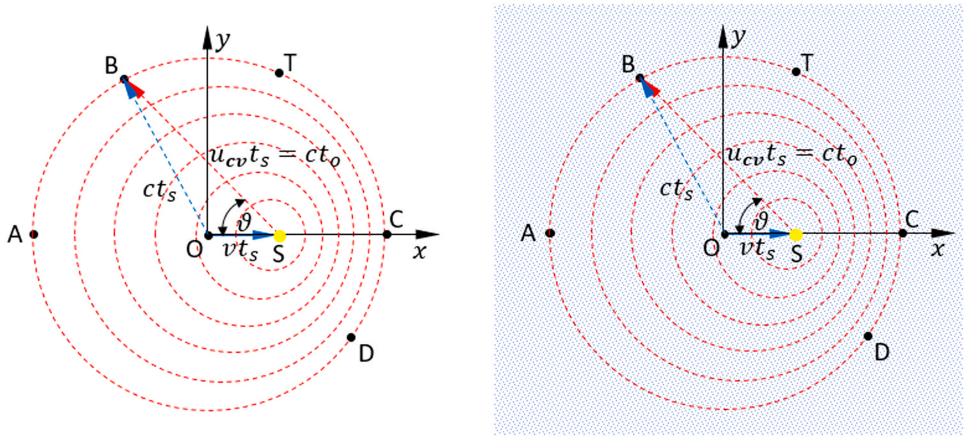

Fig. 4. Relative velocity in the DE for electromagnetic wave (*a*); relative velocity in DE for sound (*b*).

The explanation of DE for sound waves in textbooks [1–4] begins in this way: we shall take as a reference frame the body of air through which sound waves travel. This means that we shall measure the speed of a source *S* of sound waves and a detector *D* of those waves *relative to that body of air* [1].

Even Fig. 2*b* does not suggest otherwise, except that in Fig. 2*b* a frame of reference with origin at O is linked with the "body of air". But when the DE for light is treated in the same textbooks they say that the DE for light waves depends on only one velocity, the relative velocity between source and detector, as measured from the reference frame [1]. However, the same can be said of sound, because when the observer is resting, they are connected to the "body of air" – i.e. to the frame with origin at O (Fig. 2*b*). Thus, all authors who treat the DE for light saying at the very beginning that "we will take a reference frame relative to which the light source moves" [1–4, 7], have in fact not chosen a "body of air", but a "body" relative to which the waves and source move (Fig. 3), but this doesn't make any difference relative to the DE for sound. More concretely, modern physics claims that in optics the DE depends only on the relative velocity between source and observer, while in acoustics only on the relative velocity between source and medium; but this is the same thing – if we are speaking about a moving source and a resting observer – because, as is pointed out above, the reference frame of a resting observer and the reference frame of a medium represent only one (the same) reference frame. However, is it true that the DE in optics depends on the relative velocity between a source and an observer? And is it true that the DE in acoustics depends on the relative velocity between a source and a medium? If it were true, then why do we have for $v = const$ (in magnitude and direction) different observed frequencies for different observers (observers A, B, C in Fig. 2)? The truth is that the DE in both cases – optics and acoustics (Fig. 2) – depends only on one relative velocity, and that is the *relative velocity between a wave and its source*. The proof of this is set out below.

Fig. 4 once more treats the DE for electromagnetic waves (Fig. 4*a*) and for sound (Fig. 4*b*). In both cases the velocity of the wave is noted with *c* and the velocity of its source with *v*. For any position of a resting observer (A, B, C, D or T in Fig. 4*a* and *b*), he will record the same wavefront velocity (the velocity *c* in each case). But the observer records a different observed frequency f_o in the different positions even when the magnitude and direction of vector v are constant. Since an alteration of the observer's position alters the angle ϑ (Fig. 4*a* and *b*), then the dependence of the observed frequency on the observer's position can be explained by the alteration of this angle. But is there any other quantity that varies depending on the angle ϑ, when the magnitude of velocity c, magnitude of velocity v and direction of velocity v are constant?

Yes, there is. It can be seen in Fig. 4(*a* and *b*) that by moving the observer around the wavefront, the vector of velocity u_{cv} changes (in magnitude and direction). This vector represents exactly the relative velocity between the wave (with velocity c) and its source (with velocity v), and the angle ϑ is formed by the direction of the velocity of the source (v) and the direction of the observer's sight, i.e. the direction of the vector of velocity u_{cv}. Therefore, the dependence of observed frequency f_o is:

$$f_o = F_{f_o}(f_s,\ u_{cv}) \tag{1}$$

where f_s is the frequency of the source, and since:

$$u_{cv} = F_u(c, v, \vartheta) \tag{2}$$

then the functional dependence of frequency f_o can be written:

$$f_o = F_{f_o}(f_s, c, v, \vartheta) \tag{3}$$

Eqs. (1) and (3) represent the implicit form of the DE. From these equations it follows that the DE formula depends not only on the velocity of the source but also on angle ϑ. This indicates that if we want to identify the only physical quantity on which the observed frequency (the DE formula) depends, then this is the relative velocity between the wave and its source u_{cv} (Fig. 4 and Eq. (1)). The explicit form of these equations will be found below.

For a resting observer at B (Fig. 4) these two equations can be applied: [8].

$$c = \frac{n\lambda_s}{nT_s} \tag{4}$$

because from a resting source to the point of observation the total length is nT_sc or expressed by wavelength the same distance is $n\lambda_s$; and

$$u_{cv} = \frac{n\lambda_o}{nT_s} \tag{5}$$

because from a moving source to the point of observation the total length is nT_su_{cv} or expressed by wavelength the same distance is $n\lambda_o$; where n is the number of wavefronts or number of periods, λ_s is the wavelength of the source, λ_o is the observed wavelength and T_s is the emitting period of the source. From Eqs. (4) and (5) it follows:

$$f_o = f_s\frac{c}{u_{cv}}. \tag{6}$$

This equation represents the explicit form of Eq. (1). To be more explicit, the explicit form of Eq. (2) must be found. For this, using the law of cosines for the magnitude of relative velocity u_{cv}, we obtain: [8,9].

$$u_{cv} = \sqrt{c^2 - v^2 sin^2\vartheta} \quad + vcos\vartheta. \tag{7}$$

This equation represents the relative velocity between the wave and its source in the general case. For special cases (A, C and T), for $\vartheta = 0^\circ$, $\vartheta = 180^\circ$ and $\vartheta = 90^\circ$, Eq. (7) is transformed thus:

$$u_{cv}(0^\circ) = c + v \tag{8}$$

$$u_{cv}(180^\circ) = c - v \tag{9}$$

$$u_{cv}(90^\circ) = \sqrt{c^2 - v^2}. \tag{10}$$

Since the explicit form of u_{cv} has been found (Eq. (7)), the more explicit form of Eq. (6) is now:

$$f_o = f_s\frac{c}{\sqrt{c^2 - v^2 sin^2\vartheta} \quad + vcos\vartheta}. \tag{11}$$

Eqs. (6) and (11) represent the explicit form of the DE. So the observed frequency of a moving light source has been found, and from these equations it is clear that this frequency depends on the relative velocity between a wave and its source (u_{cv}). This explanation shows that the relative velocity u_{cv} is present in all DE's equations as a single and particular quantity. Expressions such as $c + v$; $c - v$; or $c + vcos\vartheta$ in DE equations represent relative velocity u_{cv}, in specific situations. The relative velocity $c + vcos\vartheta$ which appears in DE formulas, for example:

$$f_o = f_s\frac{c}{c \quad + vcos\vartheta} \tag{12}$$

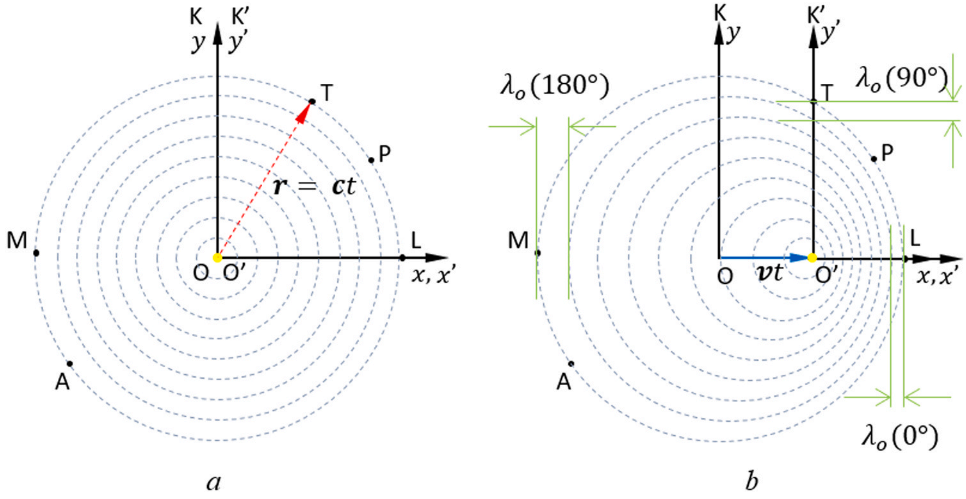

Fig. 5. Wavefronts issued from a resting source (*a*). Wavefronts issued from a moving source (*b*).

Table 1
The TDE for sound and light when the velocity of the source is $v = 100m/s$.

The velocity of source	The TDE for sound	The TDE for light
v	$f_o = f_s \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$	$f_o = f_s \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$
$100m/s$	$1,00335419f_s$	$1,000000005f_s$

is not correct, because in it the contribution of the velocity component $v\sin\vartheta$ is arbitrarily neglected [8]. Instead of the expression $c + v\cos\vartheta$ in the DE formulas which contain it, there should be the correct equation of relative velocity u_{cv} (Eq. (7)). It is understood that for very small angles Eq. (12) can also be used as an approximation of Eq. (11), but it is surprising that modern physics considers Eq. (12) to be correct and Eq. (11) non-existent. The degree of difference between Eqs. (11) and (12) and other details about these two equations can be found in the cited reference [8].

So much for relative velocities. Now to the sound wave speed's "dependence" on the reference frame. According to Feynman "… if the source of disturbance is moving, the emitted light goes through space at the same speed c. This is analogous to the case of sound, the speed of sound waves being likewise independent of the motion of the source" [10]. The same status of the velocities of light waves and sound waves can be seen also in Fig. 4 and Eq. (7). Eq. (7) is in full conformity with the principle of relativity, according to which in a relative motion there is no privileged velocity. According to this principle all velocities of Eq. (7) thus stay untouched regardless of the reference frame.

3. The "difference" in the transverse Doppler effect

The STR claims that the TDE appears only in the DE for electromagnetic waves [1–4,11–14]. We will easily prove that the TDE appears also in the DE for sound waves. Let us take two frames of reference, K and K′ (Fig. 5*a*). A sound wave source is connected to the origin of frame K′. The origins of both frames overlap and the relative velocity between them is zero (i.e. the source is resting relative to both frames). The source emits a certain frequency (f) and the wave velocity c is constant for any resting observer. Now let the source move in direction x with constant velocity v, Fig. 5*b*. The view in Fig. 5*b* is experienced from the resting frame of reference K (observers at O, A, M, T, P, L or anywhere in K). Even in this case, the wave velocity c is constant – the same for each observer, as in the case of a resting source. But, as we have pointed out in a previous section, what we can clearly see (even with the naked eye) is that the observed wavelength λ_o (or observed frequency f_o) varies depending on the observer's position. The number of wavefronts (n).

is the same in both frames. The shortest wavelength will be recorded by the resting observer at point L, and the longest by the resting observer at M; and it is obvious that a value of wavelength different from those recorded by other observers is recorded by the observer at point T, whose viewing direction is perpendicular relative to the direction of the motion of the source. So for observers at L and M the DE turns into longitudinal DE (LDE), whereas for the observer at T it's TDE. The fact that even for mechanical waves there is a TDE, namely, the fact that: $\lambda_o(0°) \neq \lambda_o(180°) \neq \lambda_o(90°)$, is more easily verified for mechanical waves than for light if we take in account only the ratio $\frac{v^2}{c^2}$, because this ratio is greater when c is the velocity of sound or water waves than when c is the velocity of light (Table 1) [15]. But the fact that the emitted frequency of a light source is much greater than the emitted frequency of a sound source makes the TDE for light more easily verified than that for sound. For example, if the frequency of sound of a train horn is 4000 Hz then, according to Table 1, the observed TDE frequency will be 4013 Hz. The difference is thus only 13 Hz. On the other hand, in the

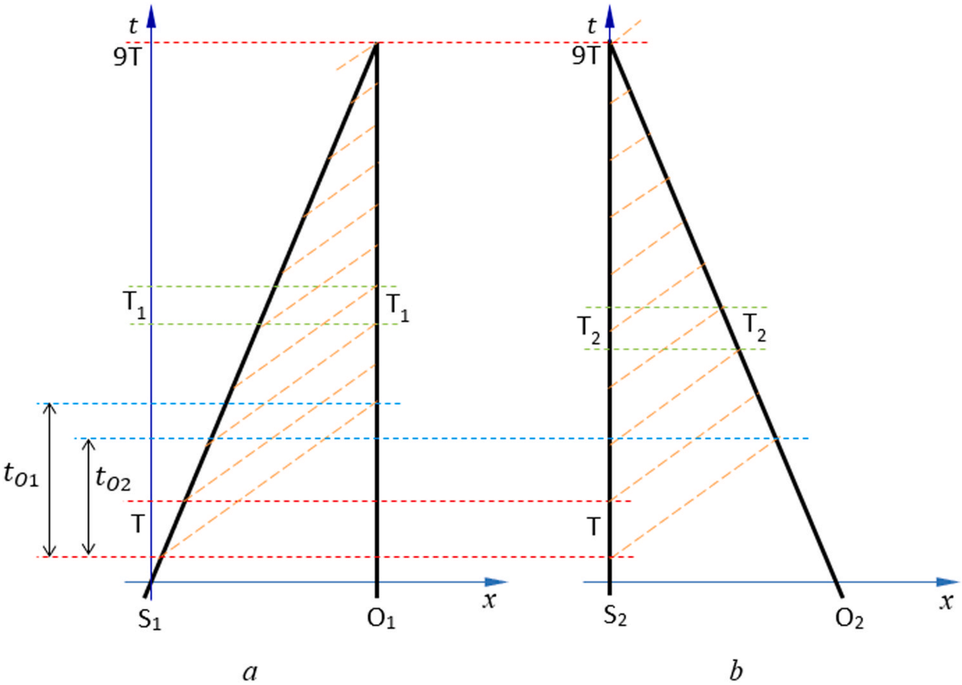

Fig. 6. Space-time diagram. The light source moves towards an observer at rest (a); an observer moves toward a light source which is at rest (b).

observed TDE frequency for blue light (650 THz), this difference is more than 3 million Hz. We should remember that in all textbooks the DE of a mechanical wave has a standard view which is like that in Fig. 5*b*.

Eq. (11) represents a general DE formula (GDE) for an arbitrary angle [8,16–19]. The GDE (11) is obtained without any approximation, without any arbitrary intrusion. Below we give a summary of the three special forms of Eq. (11): for $\vartheta = 0^\circ$, $\vartheta = 90^\circ$ and $\vartheta = 180^\circ$:

$$f_o = \left\{ \begin{array}{c} f_s \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2} sin^2\vartheta} + \dfrac{v}{c} cos\vartheta}, \text{for any angle} \quad \vartheta \\ f_s \dfrac{c}{c+v}, \text{for} \quad \vartheta = 0^\circ \\ f_s \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}, \text{for} \quad \vartheta = 90^\circ \\ f_s \dfrac{c}{c-v}, \text{for} \quad \vartheta = 180^\circ. \end{array} \right\} \tag{13}$$

As we can see, the TDE for mechanical waves was easily explained, as was the longitudinal DE. So why is the explanation of the TDE for sound and water waves absent from textbooks? Because "classical mechanics" uses Eq. (12), which claims to express the DE for an observer who receives a signal at angle ϑ from a moving source. By interpreting Eq. (12), modern textbooks come to the conclusion that the classical theory predicts no TDE [1–4,10–14]; or, in the classical theory, that for $\vartheta = 90^\circ$ the DE vanishes [1]. But, as we have pointed out (Fig. 5b, Eq. (13)) the facts say otherwise: even for $\vartheta = 90^\circ$ the DE for sound waves does not vanish, we just need to measure it. The "classical" DE formula (Eq. (12)) is derived arbitrarily; specifically, as it is pointed out above, in this equation only the contribution of the velocity component $vcos\vartheta$ is taken into account while the contribution of velocity component $vsin\vartheta$ is arbitrarily neglected.

4. The "difference" in the frequency that we observe when we move toward a source vs. the frequency that we observe when the source moves towards us

It is well-known that for sound waves the observed frequency when the source is moving towards a resting observer is different from the observed frequency when the observer is moving towards a resting source. Here, a space-time diagram shows that the same applies in the DE for light. Fig. 6*a* shows a light source (line S_1) emitting light signals (slope dashed lines) with frequency f_s, moving with velocity v towards an observer at rest (line O_1); while Fig. 6*b* shows the source at rest (line S_2) emitting light signals with the same

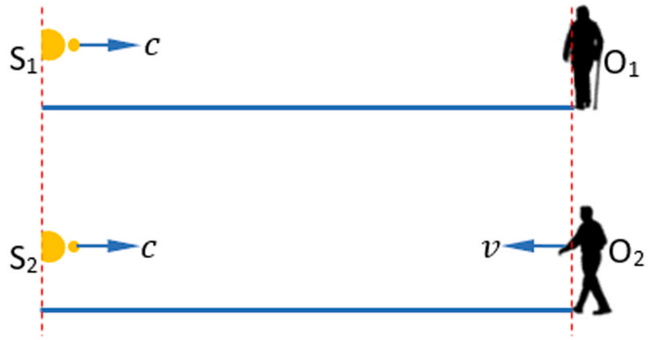

Fig. 7. Emitting and receiving of the first signal. For the first signal it is not relevant if source S_1 is moving or stationary.

frequency f_s and an observer who moves with velocity v towards the source (line O_2). Obviously, observer O_2 receives the first signal before observer O_1. This is easily explainable not only in theory, but also in practice. The distance S_1O_1 is equal to the distance S_2O_2 and the meeting point of the source and the observer will be after the same period of elapsed time in both cases, since the source (S_1) and the observer (O_2) have the same velocity v. In both cases the first signal is issued at the same time (red dashed line, bottom and parallel to the x-axis); while only observer O_2 shortens the signal's path from S_2, therefore they receive the signal before observer O_1, who is at rest (this can be seen also in Fig. 7). In fact, observer O_2 will receive all the signals in succession before observer O_1, right up to the final one which both observers will receive at the same time and which marks the meeting of the source and observer in both cases. In other words, observer O_2 starts receiving signals earlier, but receives them less often; while observer O_1 starts receiving signals later but receives them more often. From this we can draw the following conclusions: first, the receiving periods of signals from the observers are not equal ($T_1 \neq T_2$), and consequently the receiving frequencies are not equal ($f_{O1} \neq f_{O2}$); second, there is no time dilation here, because the same time elapses from the.

emission of signals in both cases ($nT = nT$); and for the reception of signals, the same amount of time elapses from the first issued signal to the reception of the latest signal (in both cases):

$$\left.\begin{aligned} nT &= nT_1 + t_{O1} \\ nT &= nT_2 + t_{O2} \end{aligned}\right\} \tag{14}$$

where t_{O1} and t_{O2} are the times within which the observers receive the first signal (Fig. 6). For these times we have:

$$\left.\begin{aligned} t_{O1} &= \frac{S_1O_1}{c} \\ t_{O2} &= \frac{S_2O_2}{c+v} \end{aligned}\right\} \tag{15}$$

Since $S_1O_1 = S_2O_2 = nTv$ from Eqs. (14) and (15) we find the frequencies for both observers:

$$\left.\begin{aligned} f_{O1} &= f_s\frac{c}{c-v} \\ f_{O2} &= f_s\frac{c+v}{c} \end{aligned}\right\} \tag{16}$$

and, as we said above, they are not equal.

According to the STR, instead of Eq. (16) only one equation is necessary [1–4,10–15], because according to this theory the observer will receive the signals from the source with equal frequency in both cases, $f_{O1} = f_{O2} = f_O$:

$$f_O = f_s\sqrt{\frac{c+v}{c-v}}. \tag{17}$$

This equation is known as the relativistic DE (RDE). According to Eq. (17) both observers will receive the first signal at the same time, and subsequently all the other signals as well; because only this scenario guarantees equal frequencies received by both observers, since the last signal must be received at the same time by both observers. There is no doubt that the last signal must be received at the same time, but as was seen in Figs. 6 and 7 it is not true that the observers receive the first signal at the same time.

Finally, for the famous question – why does the velocity of a wave source moving towards a stationary observer increase its observed frequency more in the DE than in a case when an observer is moving with the same velocity towards a stationary wave source? According to Fig. 6 the answer is: because the moving observer starts receiving signals earlier but receives them less often; while the resting observer starts receiving signals later but receives them more often.

5. Conclusion

According to modern physics there are three main differences between the Doppler effect in optics and the Doppler effect in acoustics. Using simple and reliable methods it is proved that indeed there is no difference between Doppler effect formulas for waves which involve the propagation medium (sound waves) and for waves which do not involve such a medium (electromagnetic waves). The transverse Doppler effect is proved even for sound waves. And finally we have proved that the frequency observed by a moving source and the frequency observed by a moving observer are different even for electromagnetic waves.

Declaration of Competing Interest

This manuscript has not been published and is not under consideration for publication elsewhere. Author have no conflicts of interest to disclose.

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