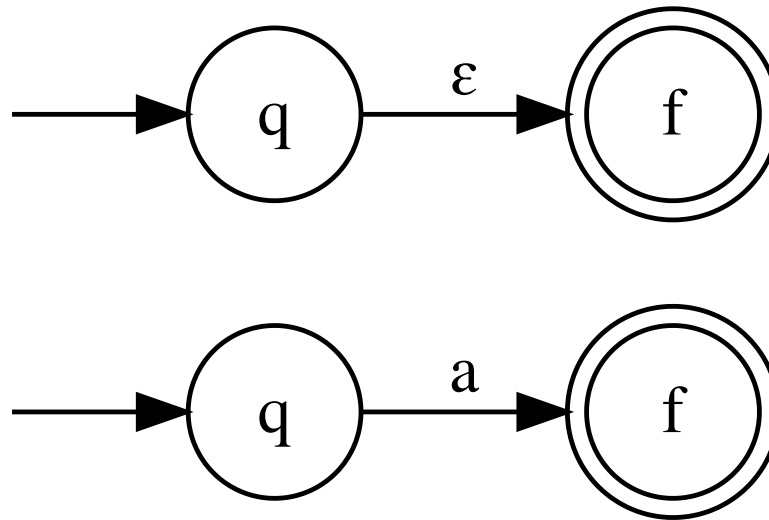


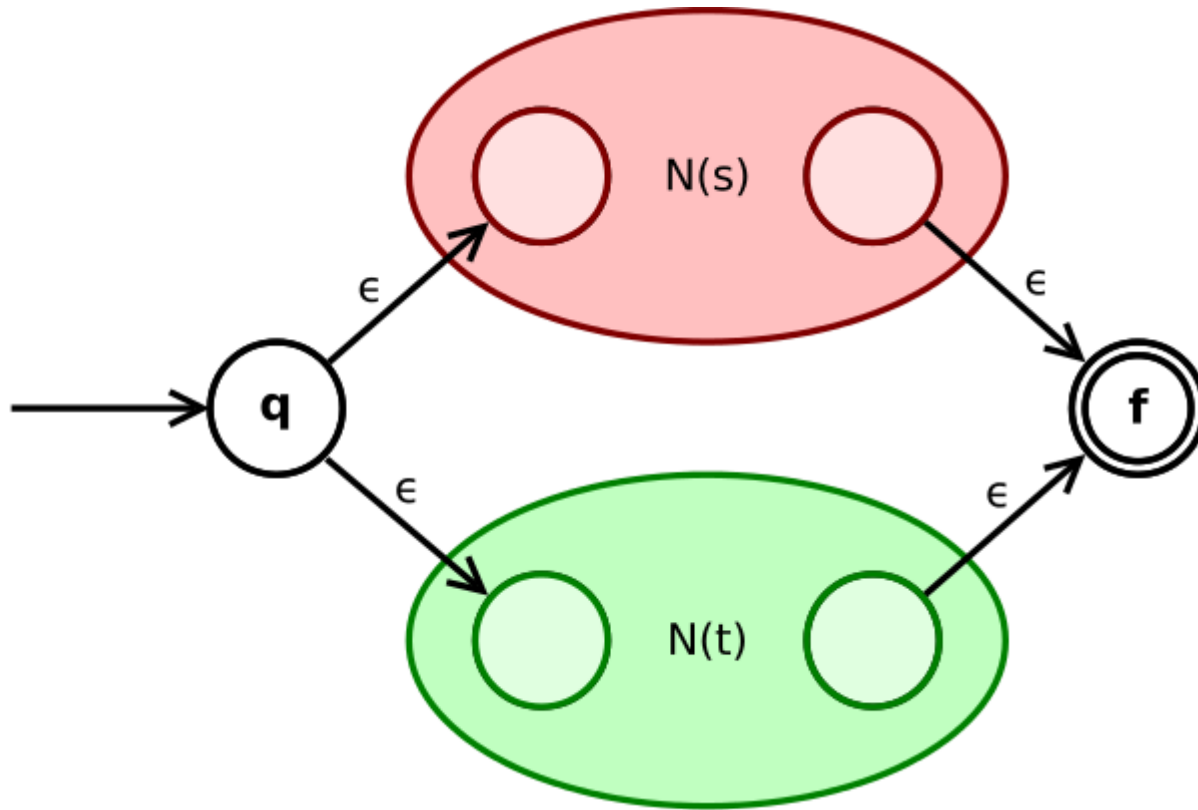
# Cas de base

Expression  $\epsilon$  et expression  $a$  où  $a$  est une lettre

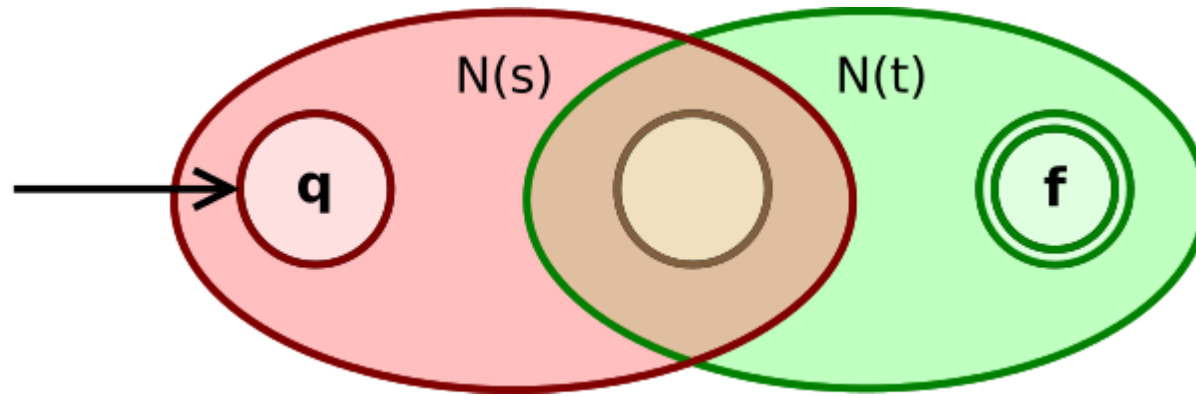


D'après Wikipédia : *Algorithme de Thompson*

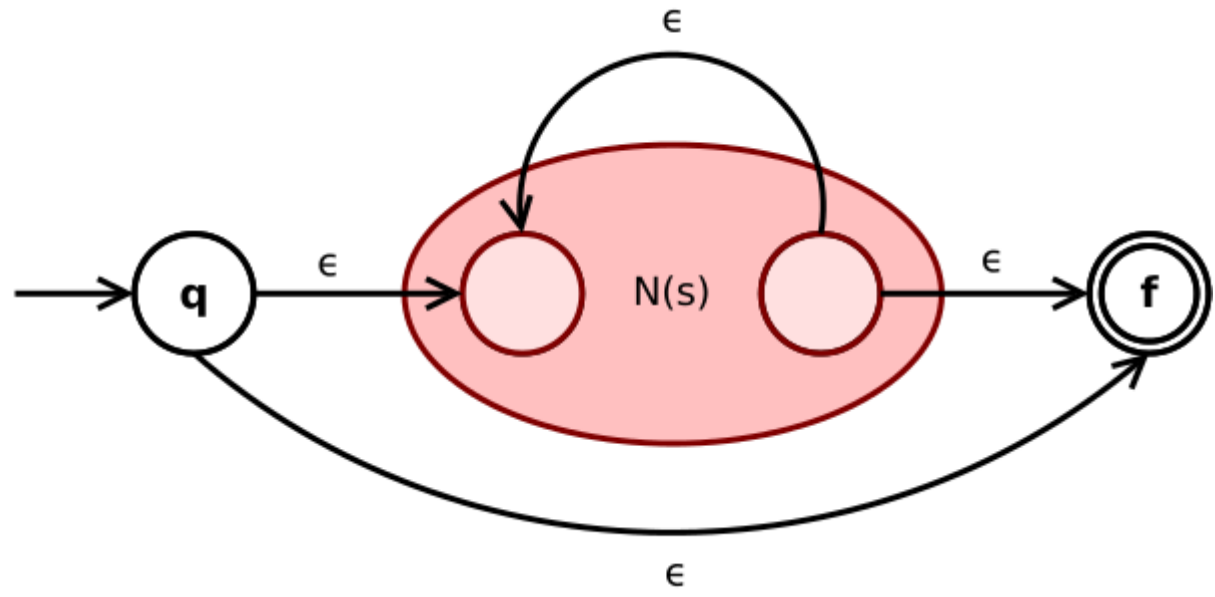
# Union (s+t)



# Concaténation s.t



# Etoile de Kleene $s^*$

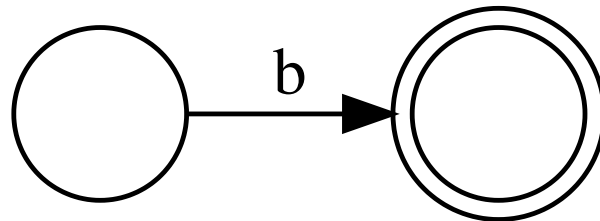
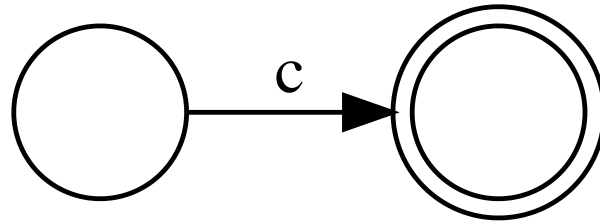


# Exercice 10

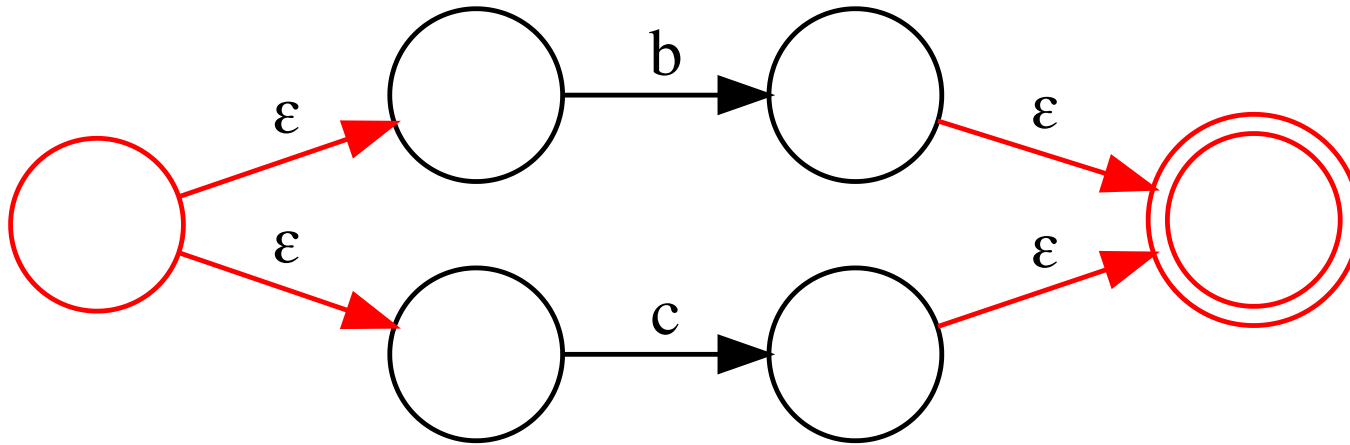
Dessiner un automate fini équivalent à :

$$a((b|c)^*|cd)^*b$$

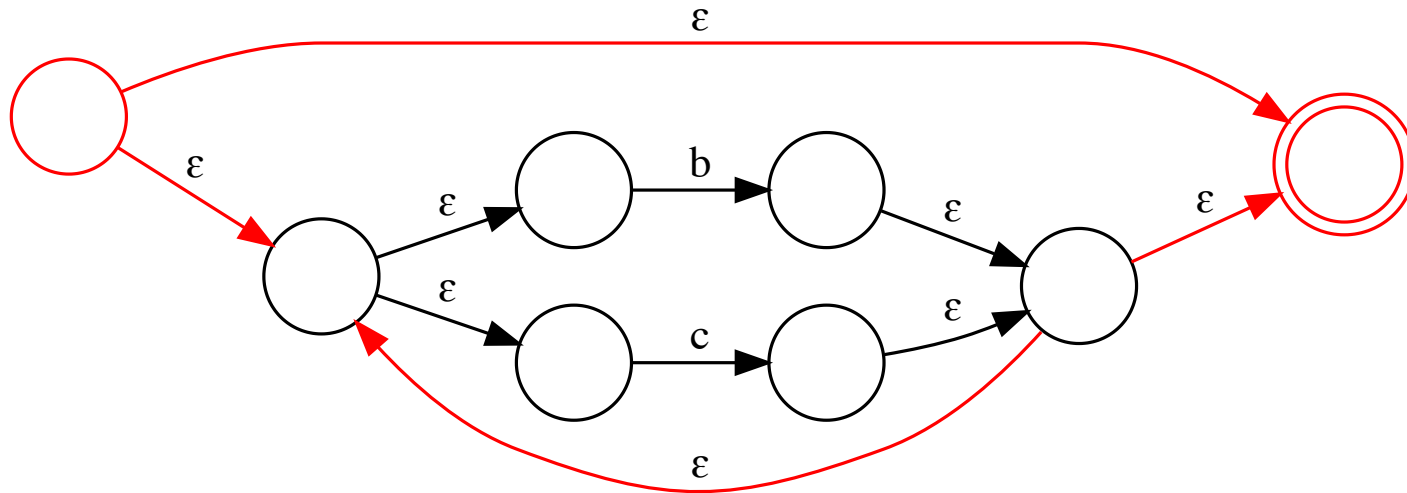
automates de b (et c)



# automate de b/c

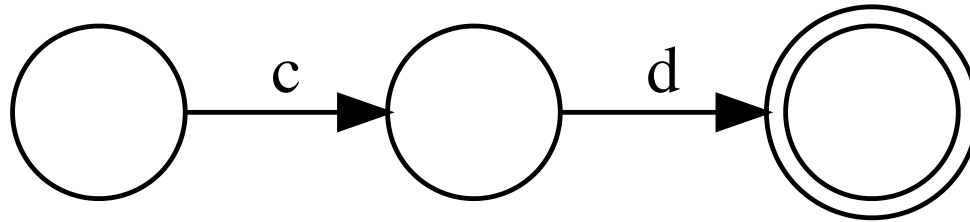


automate de  $(b|c)^*$

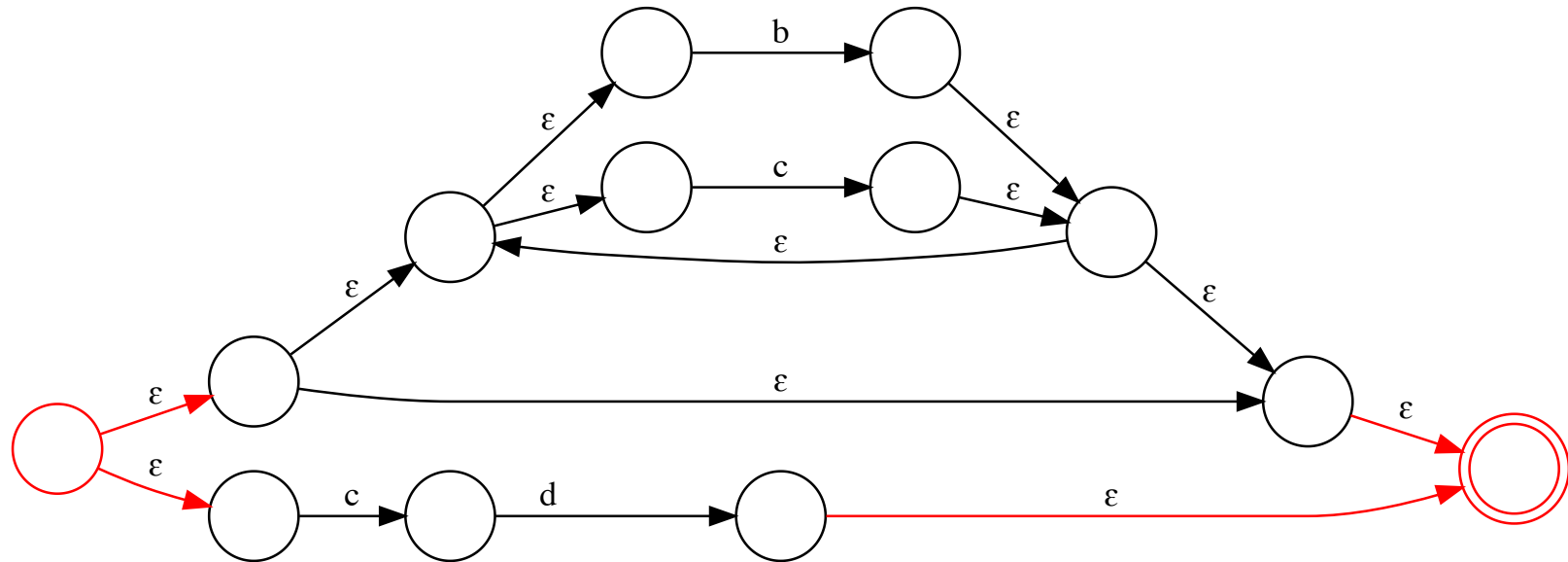




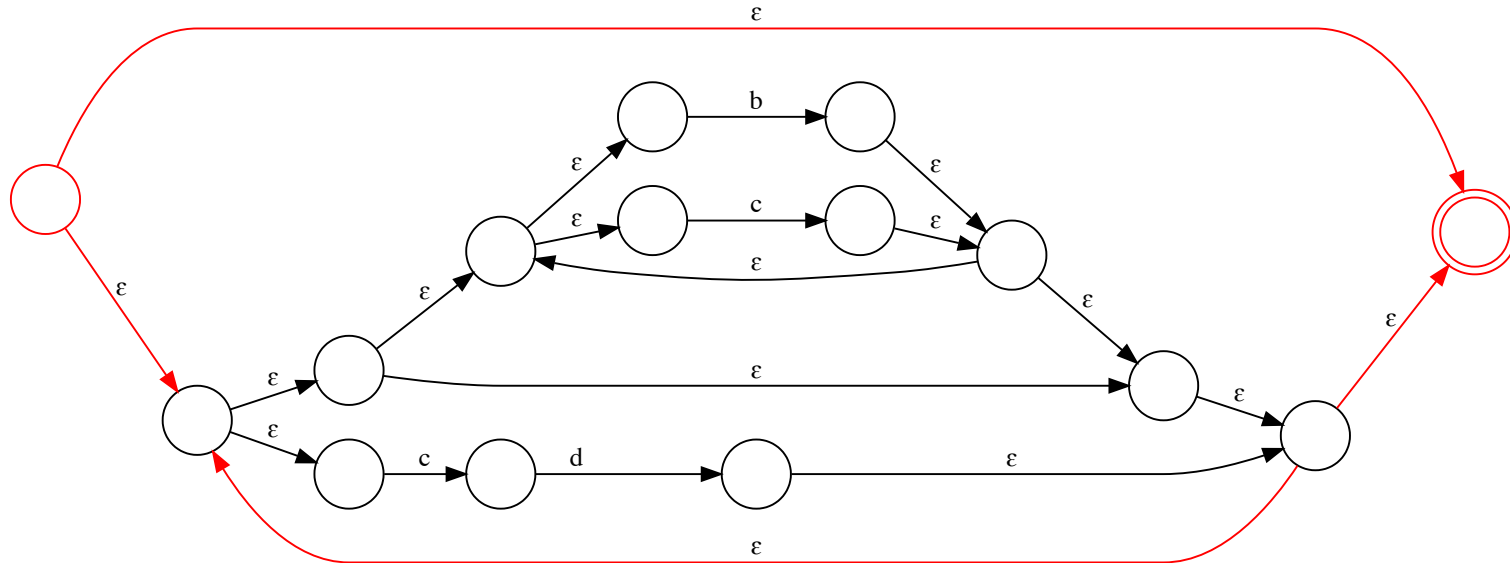
automate de cd



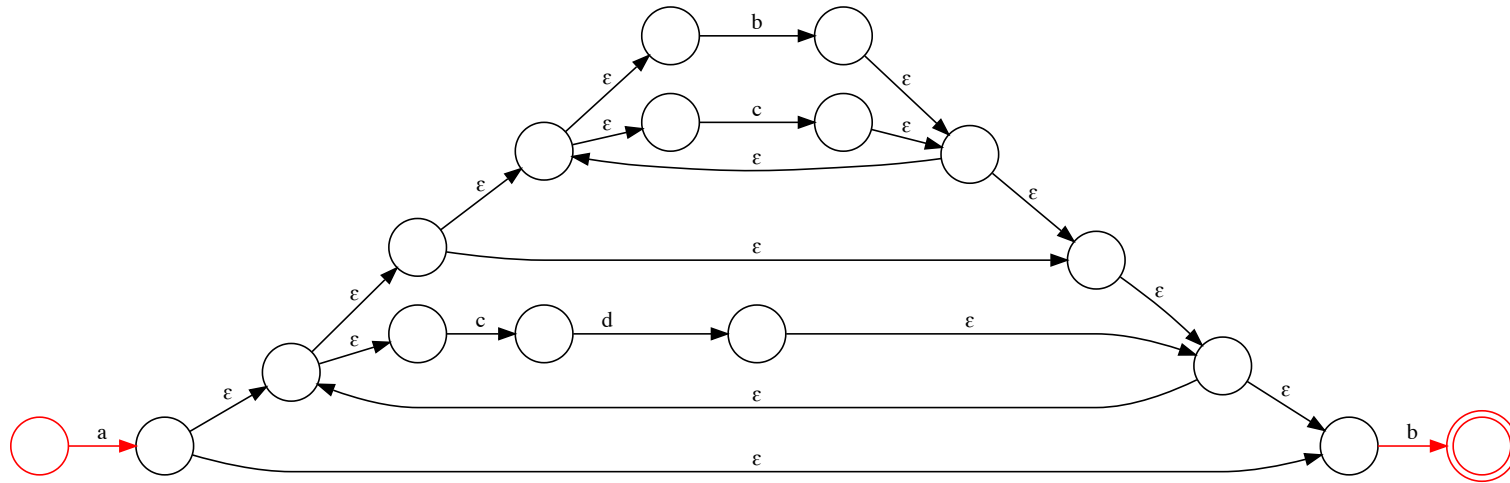
automate de  $(b|c)^*|cd$



automate de  $((blc)^*|cd)^*$



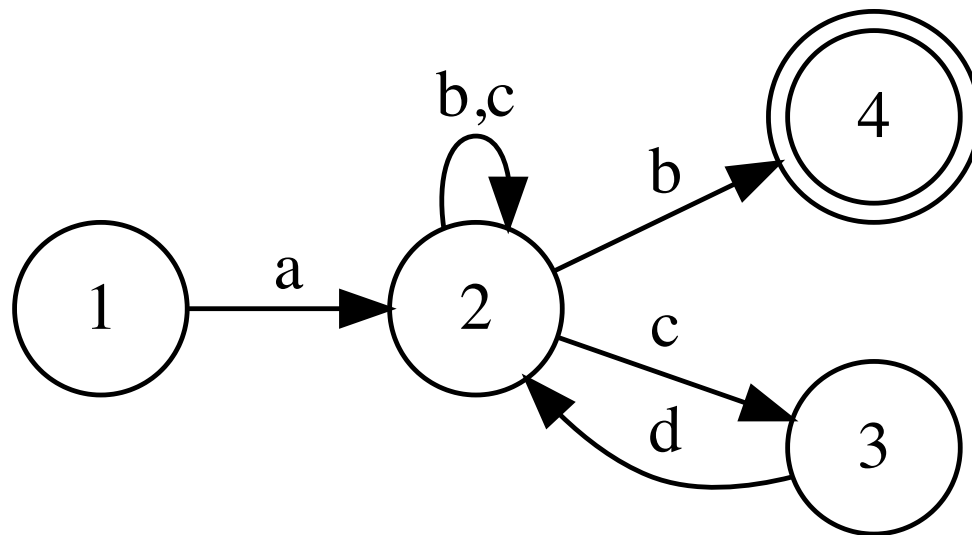
# automate de $a((b|c)^*|cd)^*b$



- supprimer les  $\epsilon$ -transitions,
- déterminer,
- minimiser

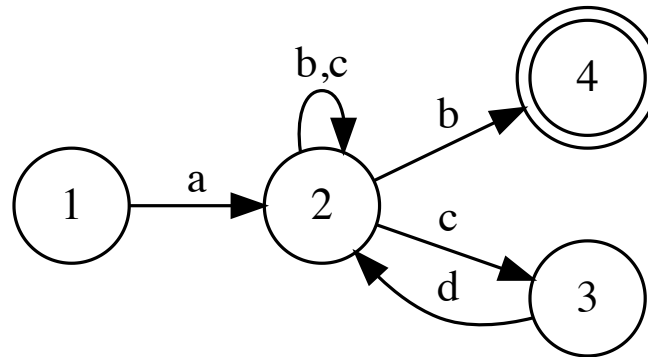
# En pratique

On part d'un automate construit à la main qui reconnaît le même langage sans  $\epsilon$ -transitions.



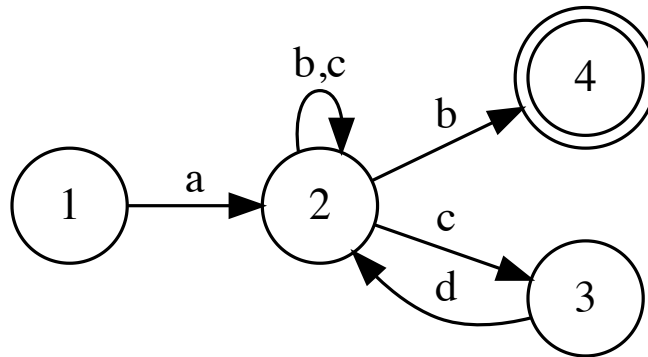
Cet automate est-il déterministe ?

# Déterminisation



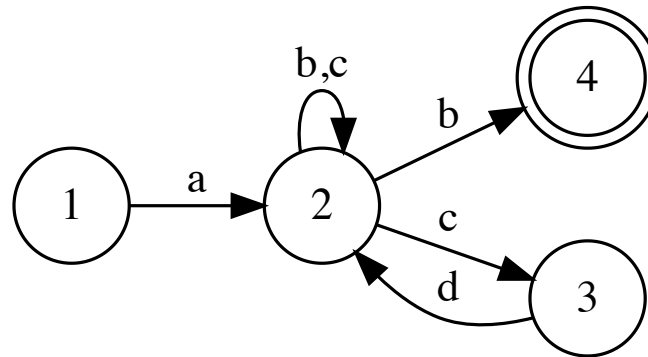
	<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>
<hr/>				
	{1}			

# Déterminisation



	a	b	c	d
{1}	{2}			
{2}				

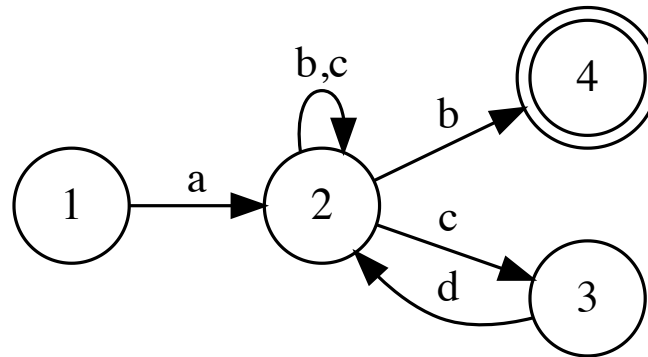
# Déterminisation



	<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>
<b>{1}</b>	<b>{2}</b>			
<b>{2}</b>		<b>{2,4}</b>	<b>{2,3}</b>	
<b>{2,4}</b>				
<b>{2,3}</b>				

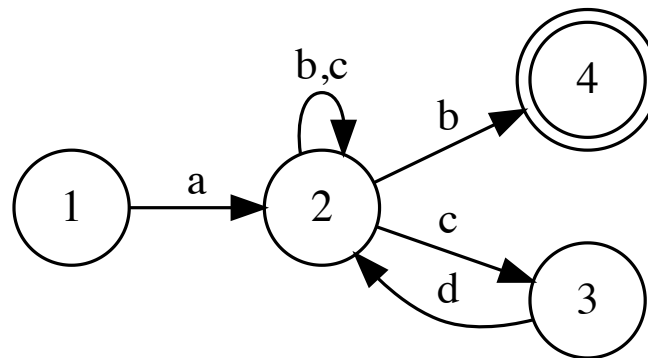


# Déterminisation



	<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>
<b>{1}</b>	<b>{2}</b>			
<b>{2}</b>		<b>{2,4}</b>	<b>{2,3}</b>	
<b>{2,4}</b>		<b>{2,4}</b>	<b>{2,3}</b>	
<b>{2,3}</b>				

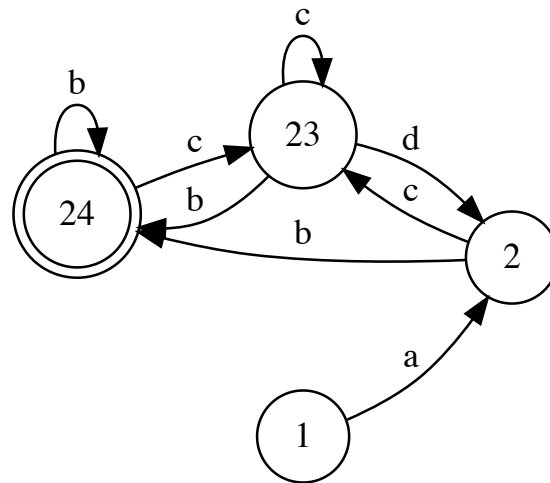
# Déterminisation



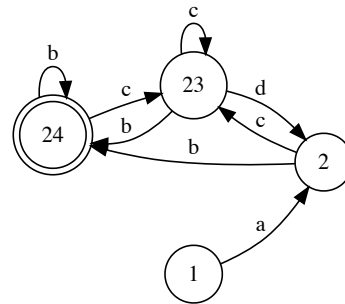
	<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>
<b>{1}</b>	<b>{2}</b>			
<b>{2}</b>		<b>{2,4}</b>	<b>{2,3}</b>	
<b>{2,4}</b>		<b>{2,4}</b>	<b>{2,3}</b>	
<b>{2,3}</b>		<b>{2,4}</b>	<b>{2,3}</b>	<b>{2}</b>

# Automate déterminisé

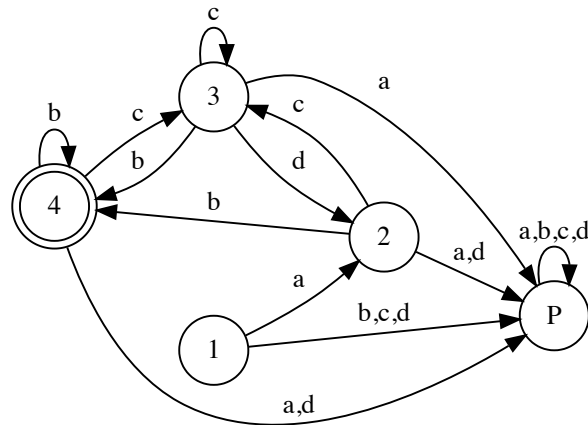
	a	b	c	d
{1}	{2}			
{2}		{2,4}	{2,3}	
{2,4}		{2,4}	{2,3}	
{2,3}		{2,4}	{2,3}	{2}



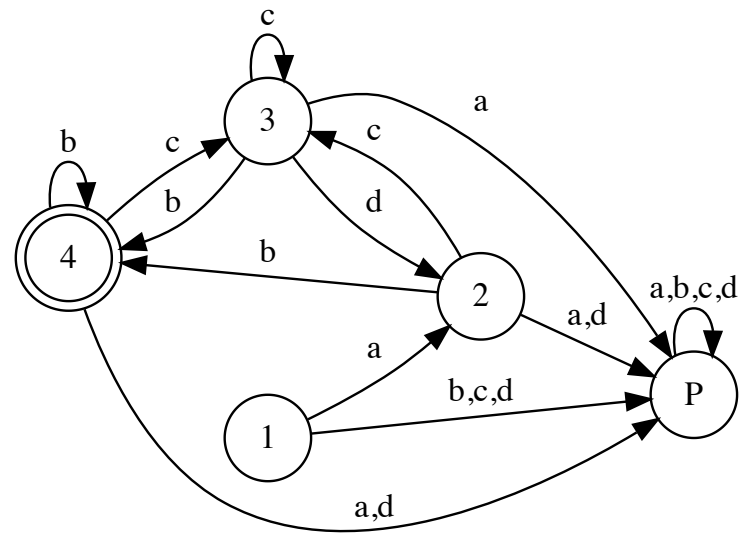
# Minimisation



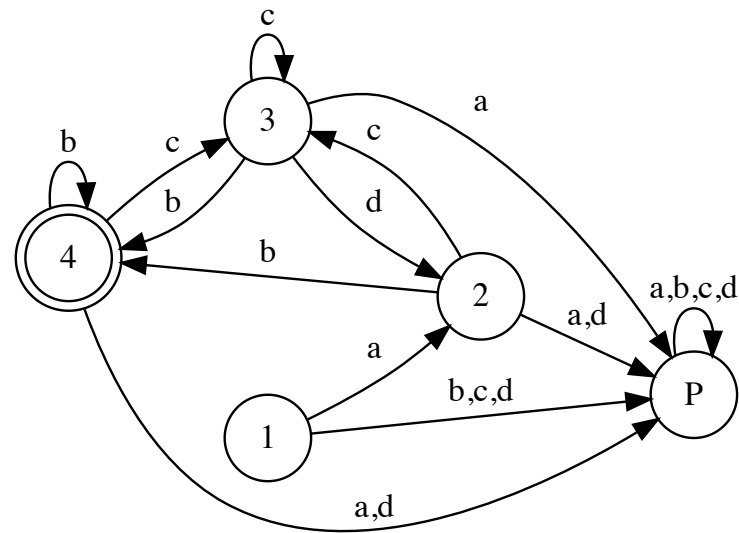
On complète l'automate avec un puits (et on renumérote).



# Minimisation

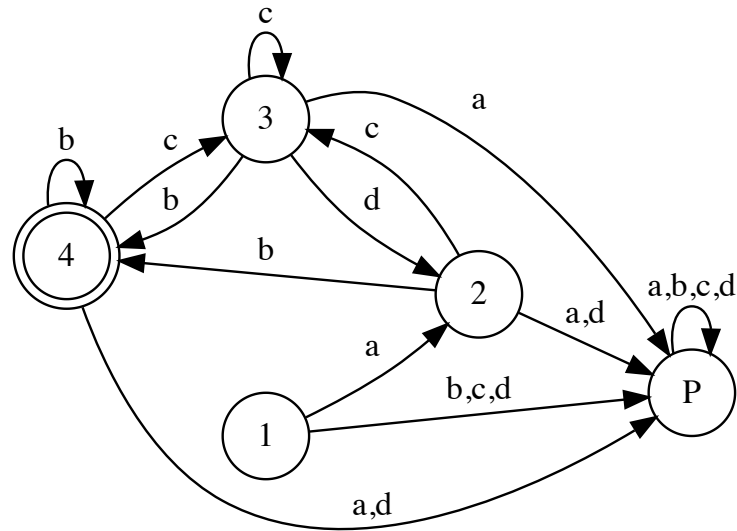


# Minimisation



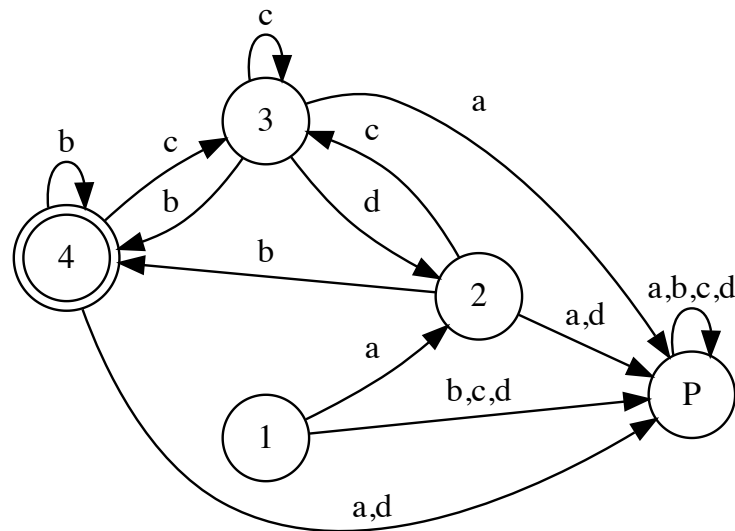
- Etape 1 : {4} {1,2,3,P}

# Minimisation



- **Etape 1** :  $\{4\} \{1,2,3,P\}$
- **Etape 2** :  $\{4\} \{2,3\} \{1,P\}$  (par **b** on sépare  $\{1,2,3,P\}$ )

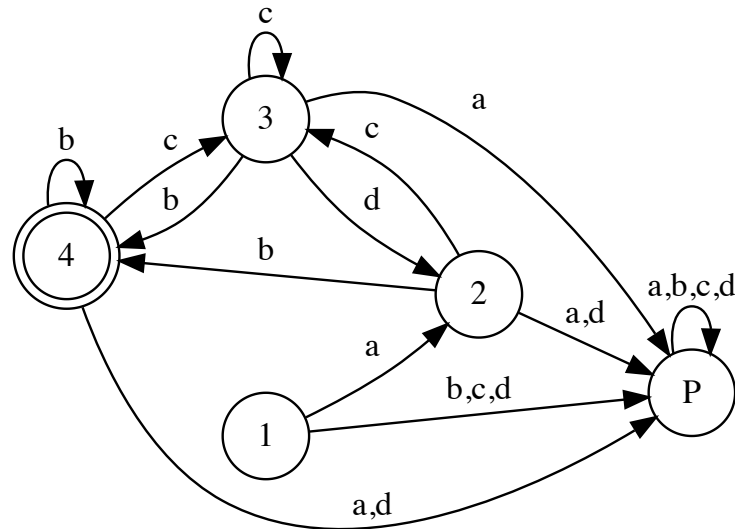
# Minimisation



- **Etape 1** :  $\{4\} \{1,2,3,P\}$
- **Etape 2** :  $\{4\} \{2,3\} \{1,P\}$  (par **b** on sépare  $\{1,2,3,P\}$ )
- **Etape 3** :  $\{4\} \{2\} \{3\} \{1,P\}$  (par **d** on sépare  $\{2,3\}$ )



# Minimisation



- **Etape 1** :  $\{4\} \{1,2,3,P\}$
  - **Etape 2** :  $\{4\} \{2,3\} \{1,P\}$  (par **b** on sépare  $\{1,2,3,P\}$ )
  - **Etape 3** :  $\{4\} \{2\} \{3\} \{1,P\}$  (par **d** on sépare  $\{2,3\}$ )
  - **Etape 4** :  $\{4\} \{2\} \{3\} \{1\} \{P\}$  (par **a** on sépare  $\{1,P\}$ )
- Quand on ne peut plus rien séparer, on enlève P, l'automate obtenu est **minimal**.

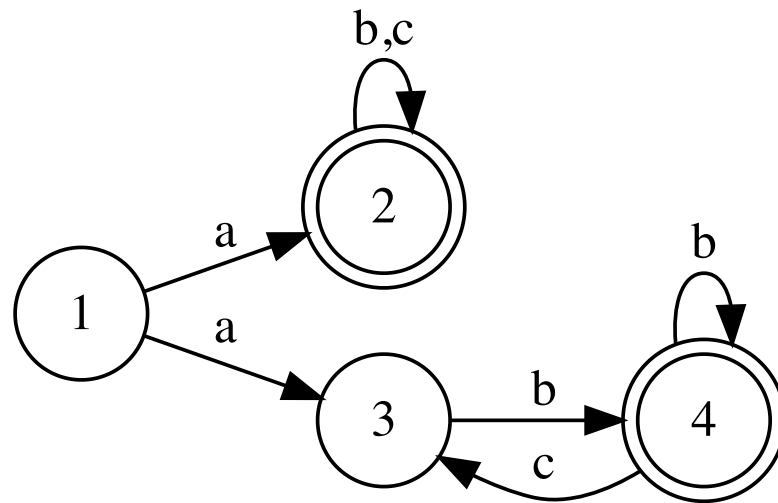
# Exercice 11

Soit l'automate fini :

$\mathbf{B} = (\{a, b, c\}, \{1, 2, 3, 4\}, \{1\}, \{2, 4\}, \{1a2, 1a3, 2b2, 2c2, 3b4, 4c3, 4b4\})$ .

- dessiner  $\mathbf{B}$
- Est-il déterministe ? Si non, le déterminer
- Minimiser  $\mathbf{B}$

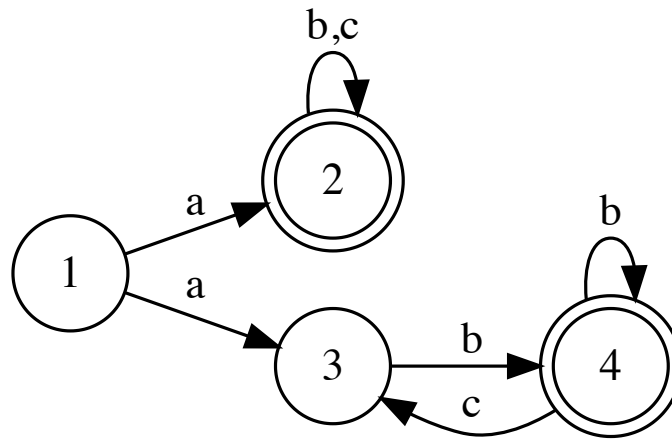
# Automate de B



$\mathbf{B} = (\{a, b, c\}, \{1, 2, 3, 4\}, \{1\}, \{2, 4\}, \{1a2, 1a3, 2b2, 2c2, 3b4, 4c3, 4b4\})$ .

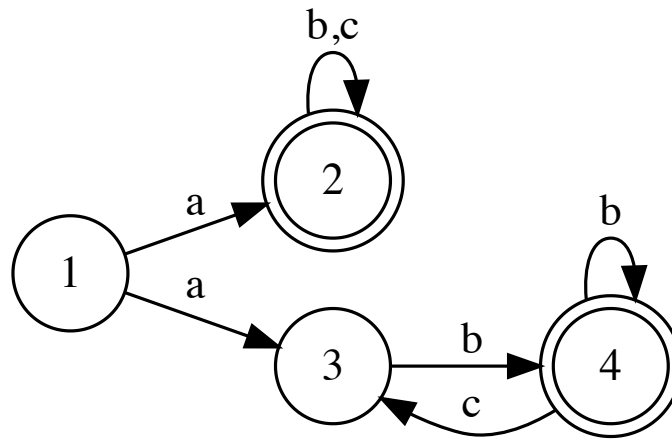
Est-il déterministe ?

# Déterminisation



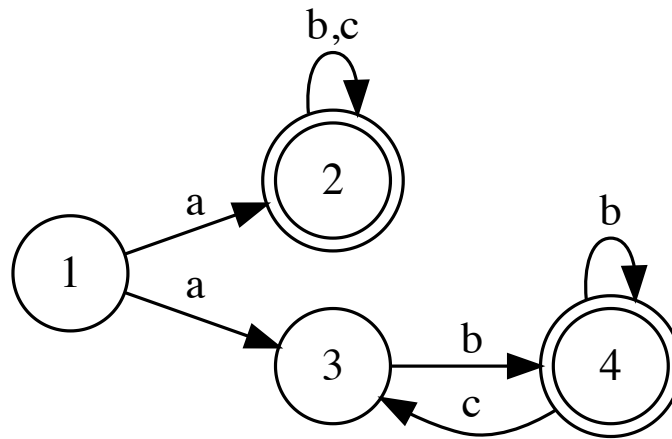
<u>a</u>	<u>b</u>	<u>c</u>
{1}		

# Déterminisation



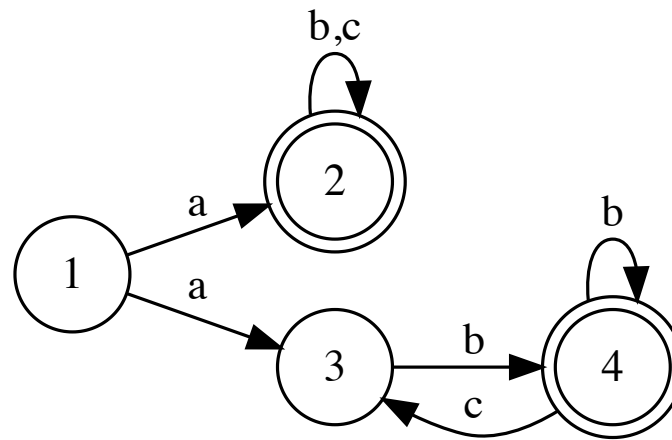
	<b>a</b>	<b>b</b>	<b>c</b>
<b>{1}</b>	<b>{2,3}</b>		
<b>{2,3}</b>			

# Déterminisation



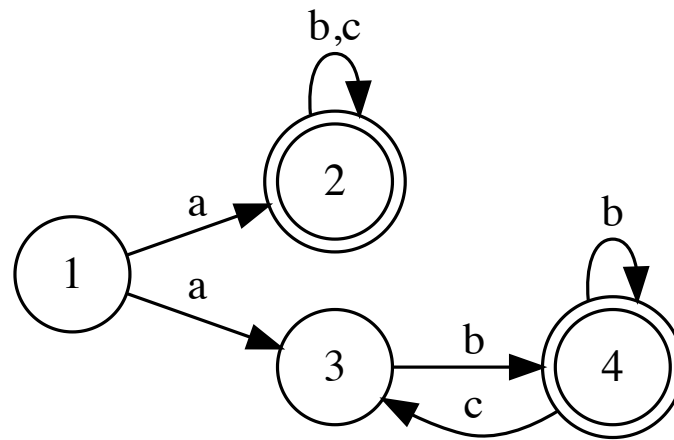
	<b>a</b>	<b>b</b>	<b>c</b>
<b>{1}</b>	<b>{2,3}</b>		
<b>{2,3}</b>		<b>{2,4}</b>	<b>{2}</b>
<b>{2,4}</b>			
<b>{2}</b>			

# Déterminisation



	<b>a</b>	<b>b</b>	<b>c</b>
<b>{1}</b>	<b>{2,3}</b>		
<b>{2,3}</b>		<b>{2,4}</b>	<b>{2}</b>
<b>{2,4}</b>		<b>{2,4}</b>	<b>{2,3}</b>
<b>{2}</b>			

# Déterminisation

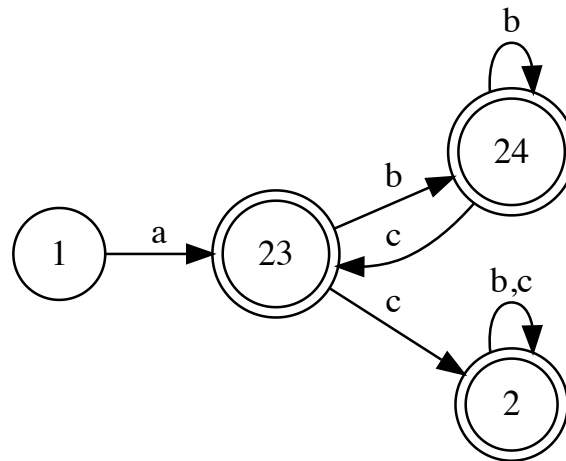


	<b>a</b>	<b>b</b>	<b>c</b>
<b>{1}</b>	<b>{2,3}</b>		
<b>{2,3}</b>		<b>{2,4}</b>	<b>{2}</b>
<b>{2,4}</b>		<b>{2,4}</b>	<b>{2,3}</b>
<b>{2}</b>		<b>{2}</b>	<b>{2}</b>

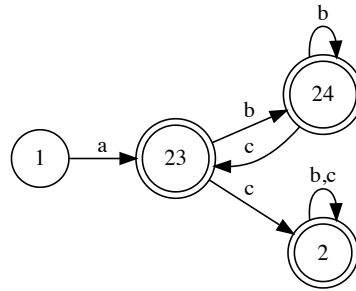


# Déterminisation

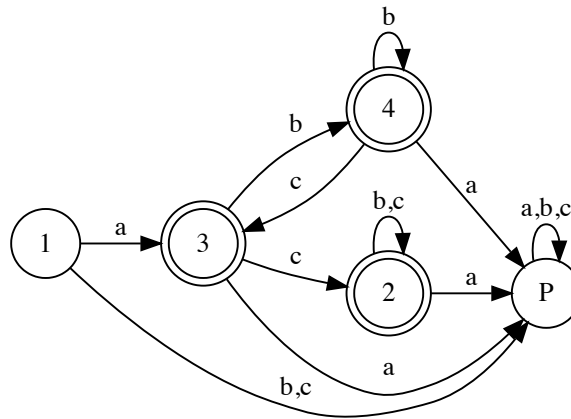
	a	b	c
{1}	{2,3}		
{2,3}		{2,4}	{2}
{2,4}		{2,4}	{2,3}
{2}		{2}	{2}



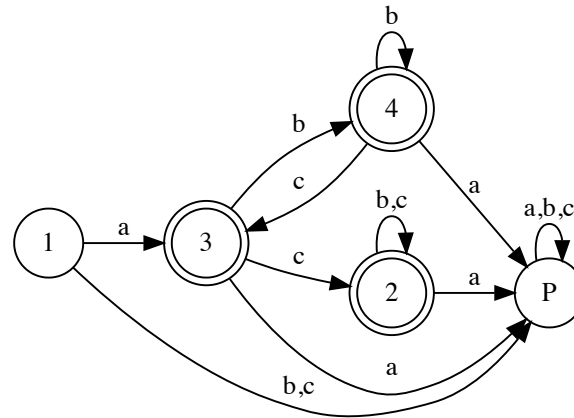
# Minimisation



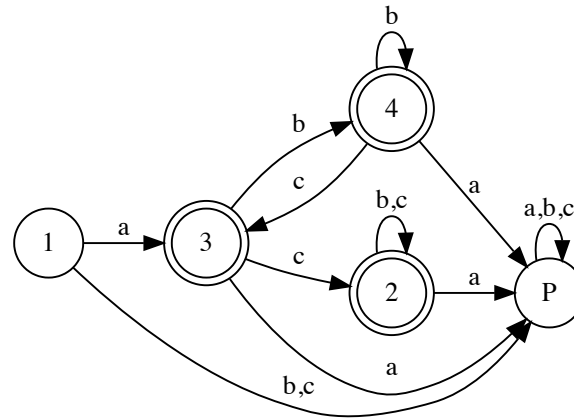
On ajoute un puits et on complète (après renumérotation).



# Minimisation

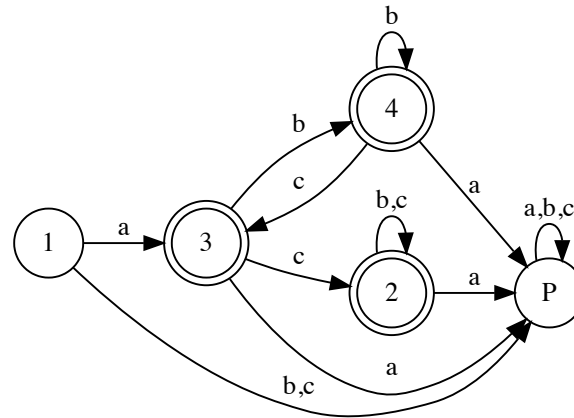


# Minimisation



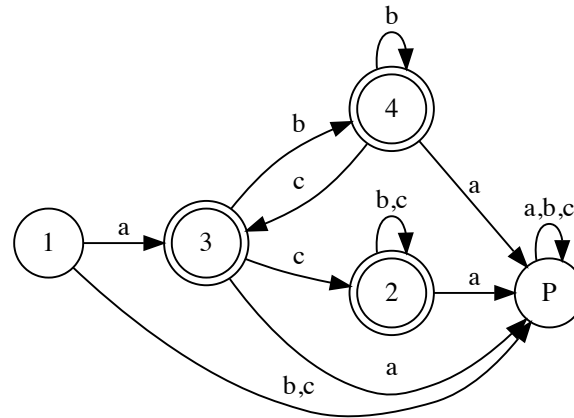
- Etape 1 : {2,3,4} {1,P}

# Minimisation



- **Etape 1** :  $\{2,3,4\} \{1,P\}$
- **Etape 2** :  $\{2,3,4\} \{1\} \{P\}$  (par **a** on sépare  $\{1,P\}$ )

# Minimisation



- **Etape 1** :  $\{2,3,4\} \{1,P\}$
- **Etape 2** :  $\{2,3,4\} \{1\} \{P\}$  (par **a** on sépare  $\{1,P\}$ )
- **Pas de séparation** pour  $\{2,3,4\}$   
On enlève P, l'automate obtenu est **minimal**.

# Minimisation

