Practical Hastings-Metropolis algorithm

Exercise 1

We want to get simulation from the distribution associated to the following density

$$f(x) = \frac{1}{Z}\exp(-x^2)\,(2+\sin(5x)+\sin(2x))\,, \quad x \in [-3,3]$$

whre Z is the normalizing constant of f.

1 Draw the graph of the unnormalized density.

2 Give an approximation of Z and with this, plot the graph of the normalized density.

3 Write a Metropolis-Hastings algorithm to simulate according to f. You can use a Gaussian random walk proposal with variance σ^2.

4 Simulate the first $50,000$ of the corresponding Markov chain.

5 Using an histogram, check that $(X_n)_{n>n_0}$ (for n_0 large enough) is a sequence of random variables which can be considered as simulation from f.

6 What is the influence of the choice of n_0 (burn-in period)?

7 Analyse the influence of the choice of σ^2.

Exercise 2 We consider the set $\underline{x} = (x_1, \ldots, x_n)$ of iid observation from a Gaussian distribution with unknown mean θ and variance equal to 2. We are interested in estimating θ using a Bayesian method. We adopt a uniform prior on $[0,5]$ for θ.

1 Give the density of the posterior distribution $\pi(\theta|\underline{x})$.

2 Propose an Metropolis-Hastings algorithm to get simulations from $\pi(\theta|\underline{x})$.

3 Use this algorithm to simulate a sample of $N = 1,000$ random variables approximately distributed from $\pi(\theta||underlinex)$ (a sample of size $n = 20$ will first have been generated with $\theta = 3$).

4 Give an approximation of $\mathbb{E}^\pi[\theta|\underline{x}]$.