Probability Formulas

1 Discrete Distributions

Uniform Distribution

Support

$$X(\Omega) = \{1, 2, \ldots, n\}$$

Probability mass function for $k \in X(\Omega)$

$$P(X = k) = \frac{1}{n}$$

Expected value

$$\mathbb{E}(X) = \frac{n+1}{2}$$

Variance

$$\mathbb{V}(X) = \frac{n^2 - 1}{12}$$

Bernoulli Distribution

Support

$$X(\Omega) = \{0, 1\}$$

Parameter

$$p \in [0, 1]$$

Probability mass function

$$P(X = 1) = p$$

Expected value

$$\mathbb{E}(X) = p$$

Variance

$$\mathbb{V}(X) = p(1 - p)$$

Binomial Distribution

Support

$$X(\Omega) = \{0, 1, \ldots, n\}$$

Parameter

$$p \in [0, 1]$$

Probability mass function for $k \in X(\Omega)$

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

Expected value

$$\mathbb{E}(X) = np$$

Variance

$$\mathbb{V}(X) = np(1 - p)$$

Poisson Distribution

Support

$$X(\Omega) = \mathbb{N}$$

Parameter

$$\lambda > 0$$

Probability mass function for $k \in X(\Omega)$

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Expected value

$$\mathbb{E}(X) = \lambda$$

Variance

$$\mathbb{V}(X) = \lambda$$

Geometric Distribution (Time to first success)

Support

$$X(\Omega) = \mathbb{N}^*$$

Parameter

$$p \in [0, 1]$$

Probability mass function for $k \in X(\Omega)$

$$P(X = k) = (1 - p)^{k-1} p$$

Expected value

$$\mathbb{E}(X) = \frac{1}{p}$$

Variance

$$\mathbb{V}(X) = \frac{1 - p}{p^2}$$

Negative Binomial Distribution (Number of failures before k successes)

Support

$$X(\Omega) = \mathbb{N}$$

Parameter

$$p \in [0, 1]$$

$$k \in \mathbb{N}^*$$

Probability mass function for $x \in X(\Omega)$

$$P(X = x) = \binom{x}{k + x - 1} p^k (1 - p)^x$$

Expected value

$$\mathbb{E}(X) = \frac{k(1 - p)}{p}$$

Variance

$$\mathbb{V}(X) = \frac{k(1 - p)}{p^2}$$

2 Continuous Distributions

Uniform Distribution

Support

$$X(\Omega) = [a, b]$$

Parameter

$$a < b$$

Probability density function for $x \in X(\Omega)$

$$f(x) = \frac{1}{b-a}$$

Expected value

$$\mathbb{E}(X) = \frac{a+b}{2}$$

Variance

$$\mathbb{V}(X) = \frac{(b-a)^2}{12}$$

Normal Distribution

Support

$$X(\Omega) = \mathbb{R}$$

Parameter

$$\mu \in \mathbb{R}$$
$$\sigma^2 > 0$$

Probability density function for $x \in X(\Omega)$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Expected value

$$\mathbb{E}(X) = \mu$$

Variance

$$\mathbb{V}(X) = \sigma^2$$

Exponential Distribution

Support

$$X(\Omega) = \mathbb{R}_+^*$$

Parameter

$$\lambda > 0$$

Probability density function for $x \in X(\Omega)$

$$f(x) = \lambda e^{-\lambda x}$$

Expected value

$$\mathbb{E}(X) = \frac{1}{\lambda}$$

Variance

$$\mathbb{V}(X) = \frac{1}{\lambda^2}$$

Log-normal Distribution

Support

$$X(\Omega) = \mathbb{R}_+^*$$

Parameter

$$\mu \in \mathbb{R}$$

$$\sigma^2 > 0$$

Probability density function for $x \in X(\Omega)$

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\log x - \mu)^2}{2\sigma^2}\right)$$

Expected value

$$\mathbb{E}(X) = \exp\left(\mu + \frac{\sigma^2}{2}\right)$$

Variance

$$\mathbb{V}(X) = \left[\exp(\sigma^2) - 1\right] \exp(2\mu + \sigma^2)$$

Gamma Distribution

Support

$$X(\Omega) = \mathbb{R}_+^*$$

Parameter

$$\alpha > 0$$

$$\beta > 0$$

Probability density function for $x \in X(\Omega)$

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} \exp^{-\beta x}$$

Expected value

$$\mathbb{E}(X) = \frac{\alpha}{\beta}$$

Variance

$$\mathbb{V}(X) = \frac{\alpha}{\beta^2}$$

Chi-squared Distribution

Support

$$X(\Omega) = \mathbb{R}_+^*$$

Parameter

$$n \in \mathbb{N}^*$$

Probability density function for $x \in X(\Omega)$

$$f(x) = \frac{1}{2^{n/2}\Gamma(n/2)} x^{n/2-1} \exp^{-x/2}$$

Expected value

$$\mathbb{E}(X) = n$$

Variance

$$\mathbb{V}(X) = 2n$$

Student's t-distribution

Support

$$X(\Omega) = \mathbb{R}$$

Parameter

$$\nu > 0$$

Probability density function for $x \in X(\Omega)$

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

Expected value

$$\mathbb{E}(X) = 0, \quad \text{if } \nu > 1$$

Variance

$$\mathbb{V}(X) = \frac{\nu}{\nu - 2}, \quad \text{if } \nu > 2$$

Cauchy Distribution

Support

$$X(\Omega) = \mathbb{R}$$

Parameter

$$a > 0$$

Probability density function for $x \in X(\Omega)$

$$f(x) = \left(\frac{a}{\pi(a^2 + x^2)}\right)$$

Expected value

$$\mathbb{E}(X) \text{ does not exist}$$

Variance

$$\mathbb{V}(X) \text{ does not exist}$$

Beta Distribution

Support

$$X(\Omega) =]0, 1[$$

Parameter

$$\alpha > 0$$

$$\beta > 0$$

Probability density function for $x \in X(\Omega)$

$$f(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1 - x)^{\beta-1}$$

Expected value

$$\mathbb{E}(X) = \frac{\alpha}{\alpha + \beta}$$

Variance

$$\mathbb{V}(X) = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$$