Practical Acceptance-Rejection algorithm

Jean-Michel Marin

We are interested in the random variable X defined on $\mathbb{R}$ with density p such that

$$p(x) \propto \tilde{p}(x)$$

$$\tilde{p}(x) = \left[0.7\exp(-x^2/2) + 0.3\exp(-x^2/2 + 7(x-7/2))\right] .$$

Let $c = \displaystyle\int_{\mathbb{R}} \tilde{f}(x)\mathrm{d}x$ and then

$$p(x) = \frac{\tilde{p}(x)}{c} .$$

We want to generate a sample from f by acceptance-rejection using the instrumental distribution q. We have

$$\frac{\tilde{p}(x)}{q(x)} \leq \tilde{k} .$$

1 Which of the following couple $(q, \tilde{k})$ are valid?

1. $\mathcal{N}(2.1, 3^2)$ and $\tilde{k} = 11$;
2. $\mathcal{S}\mathrm{t}(3)$ and $\tilde{k} = 300$;
3. $\mathcal{C}\mathrm{au}(0, 1)$ and $\tilde{k} = 40$.

2 Which of the following $(q, \tilde{k})$ is the most efficient in terms of simulation numbers?

1. $\mathcal{N}(2.1, 3^2)$ and $\tilde{k} = 11$;
2. $\mathcal{S}\mathrm{t}(3)$ and $\tilde{k} = 300$;
3. $\mathcal{C}\mathrm{au}(0, 1)$ and $\tilde{k} = 40$.

3 We finally choose to use for $q \sim \mathcal{N}(2.1, 3^2)$ and $\tilde{k} = 11$. Using this couple, simulate an $100,000$-sample from p. Evaluate the acceptance rate.