University of Montpellier 2024-2025
Faculty of Sciences
HAX918X

Bayesian estimation

Exercise 1

We consider a real random variable following a Gaussian distribution $\mathcal{N}(\theta, 1)$. We assume that the parameter $\theta \sim \mathcal{N}(0, 1)$. We would like to estimate θ. Determine the Bayes estimator of θ associated with the weighted quadratic loss function $L_2(\theta, a) = \theta^2 (\theta - a)^2$.

Exercise 2

The same binary $\theta \in \{0, 2\}$ information is transmitted 2 consecutive times to a receiver through a transmission channel. These two items of information are noise of variance 1. The received message is stored in the vector $z = (z_1, z_2)$ with z_1 and z_2 two independent random variables distributed according to Gaussian distributions with mean θ and variance 1. The problem is to find the transmitted symbol θ from the received from the received message $z = (z_1, z_2)$. We assume that $\mathbb{P}(\theta = 0) = \mathbb{P}(\theta = 2) = 1/2$.

1 Give the posteriori distribution of θ.

2 Calculate the Bayesian estimator associated with the loss function

$$L(\theta, d) = \begin{cases} 0 & \text{si} \quad \theta = d \\ 2 & \text{si} \quad \theta = 0 \quad \text{et} \quad d = 2 \\ 1 & \text{si} \quad \theta = 2 \quad \text{et} \quad d = 0 \end{cases} .$$

Exercise 3

We consider a random variable x following the probability distribution $\mathcal{B}(n, \theta)$. We place ourselves in a Bayesian context and consider the unknown parameter $\theta \in [0, 1]$ as a random variable. variable.

1 Determine the Jeffreys prior distribution of θ, $\pi_1(\theta)$. Calculate the Bayesian estimator of θ corresponding to the mean of the posterior distribution of θ.

2 We consider a new a priori probability distribution $\pi_2(\theta) = \mathbf{1}_{[0,1]}(\theta)$. (uniform distribution on $[0, 1]$). Calculate the new Bayesian estimator of θ corresponding to the mean of the posterior distribution.