Additional work on multiple linear regression

Exercise 1

Consider the model

$$y_i = bx_i + u_i$$
, $\mathbb{E}[u_i] = 0$, $\mathbb{E}[u_i^2] = \sigma^2$, $\mathbb{E}[u_iu_j] = 0$

where x_i is a scalar.

Give the least squares estimator \hat{b} . Let $b^{\#} = \sum y_i / \sum x_i$ be the estimator.

Compare these two estimators by calculating their and their variance (check that the BLUE property applies: the variance of \hat{b} is less than that of $b^{\#}$).

Exercise 2

We regress y on two explanatory variables x and z, i.e. X = (1, x, z). There are n individuals in all, we obtained the following result:

$$X^T X = \left(\begin{array}{ccc} 5 & 3 & 0 \\ 3 & 3 & 1 \\ 0 & 1 & 1 \end{array}\right)$$

1. What is the value of n? What is the empirical correlation between x and z?

Linear regression provides the results:

$$y = -1 + 3x + 4z + \hat{u}$$
, $RSS = 3$.

- 2 What is the value of \bar{y} ?
- 3 Calculate $\|\hat{y}\|^2$ and deduce ESS, TSS and the coefficient of determination R^2 .

Exercise 3

Consider the regression model

$$y_i = ax_i + u_i, \quad i = 1, \dots N$$

with $E[u_i] = 0$, $\mathbb{V}(u_i) = \sigma_i^2$, $\mathbb{C}(u_i, u_j) = 0$, $i \neq j$.

 x_i and a are scalars. Give the expression for the OLS and GLS estimators of a and compare their variance.