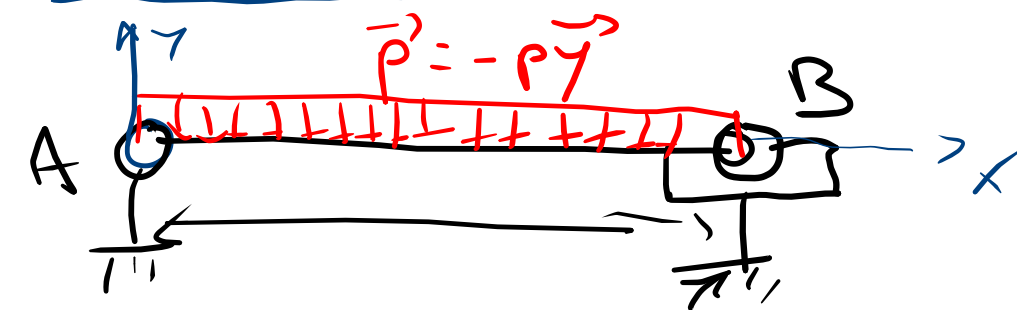


Exercice 3:



PFS appliquée à la poutre en A:

$$X_A = 0$$

$$Y_A = \frac{pL}{2} \quad Y_B = \frac{pL}{2}$$

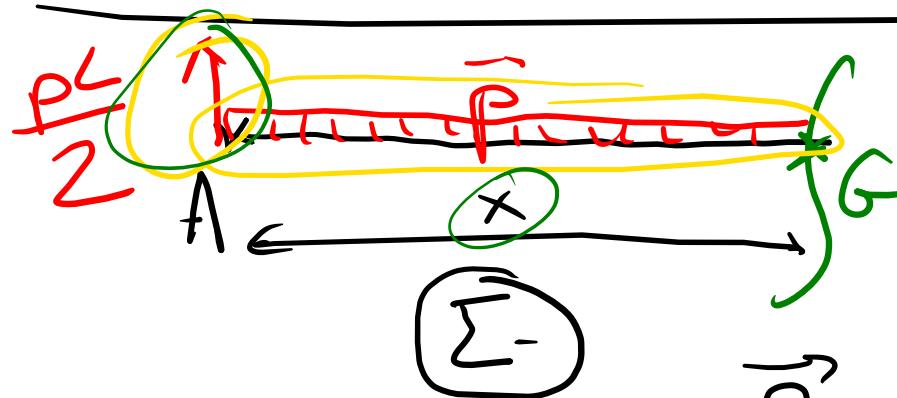
B.A.M.E:

$$\left\{ \sum_A^{\text{pivot}} \right\} = \left\{ \begin{array}{l} \vec{R}_A = X_A \vec{x} + Y_A \vec{y} \\ \vec{\Gamma}(A, \vec{\mathcal{L}}_{\text{pivot}}) \cdot \vec{z} = 0 \end{array} \right\}_A$$

$$\left\{ \sum_B^{\text{lin anul}} \right\} = \left\{ \begin{array}{l} \vec{R}_B = Y_B \vec{y} \\ \vec{\Gamma}(B, \vec{\mathcal{L}}_B) = \vec{0} \end{array} \right\}_B$$

$$\left\{ \sum_P \right\} = \left\{ \begin{array}{l} \vec{R}_P = -pL \vec{y} \\ \vec{\Gamma}(A, \vec{P}) = -\frac{pL^2}{2} \vec{z} \end{array} \right\}_A \quad (\text{voir démo Exo 2})$$

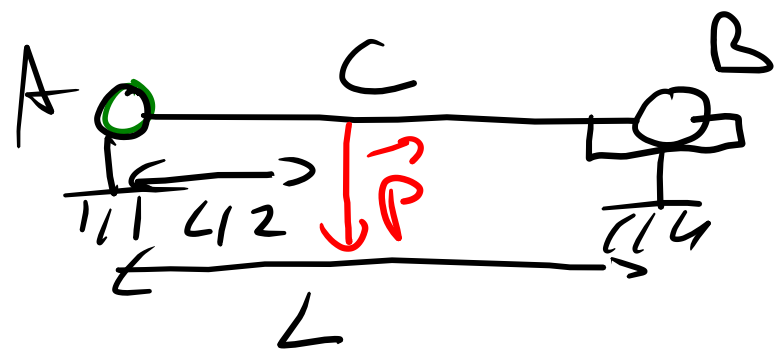
Torsion des efforts de cohésion: $0 < x < L$



$$\left\{ \sum_{P \rightarrow \Sigma} \right\}_G = \left\{ \begin{array}{l} \vec{R}_P = -p x \vec{y} \\ \vec{\Gamma}(G, \vec{P} \rightarrow \vec{\mathcal{L}}_-) = -\frac{p x^2}{2} \vec{z} \end{array} \right\}_G \quad (\text{voir démo Exo 2})$$

$$\left\{ \sum_{Ext \rightarrow \Sigma} \right\} = \left\{ \begin{array}{l} \vec{R}_{Ext \rightarrow \Sigma} = \frac{pL}{2} \vec{y} - p x \vec{y} = p \left(\frac{L}{2} - x \right) \vec{y} \\ \vec{\Gamma}(G, Ext \rightarrow \Sigma) = -\frac{pL}{2} x \vec{z} + \frac{p x^2}{2} \vec{z} \end{array} \right\}_G$$

$$= \left\{ \sum_{coh} \right\}_G = \left\{ \begin{array}{l} p \left(x - \frac{L}{2} \right) \vec{y} \\ \frac{p}{2} x (L - x) \vec{z} \end{array} \right\}_G$$



B.A.T.E.: $\{ \mathcal{L}_A \} = \left\{ \begin{array}{l} \vec{R}_A \\ \vec{M}(A, \mathcal{L}_A) = \vec{0} \end{array} \right\}_A$ $\{ \mathcal{L}_B \} = \left\{ \begin{array}{l} \vec{R}_B \\ \vec{M}(B, \mathcal{L}_B) = \vec{0} \end{array} \right\}_B$

$\{ \mathcal{L}_P \} = \left\{ \begin{array}{l} \vec{P} \\ \vec{M}(C, \vec{P}) = \vec{0} \end{array} \right\}_C$

PFS: $\rightarrow X_A = 0; Y_A = \frac{P}{2}; Y_B = \frac{P}{2}$

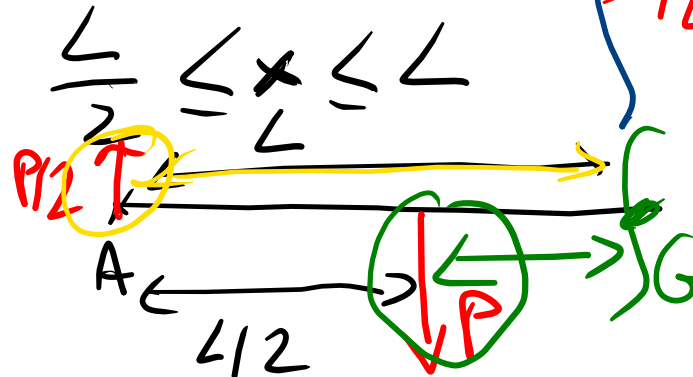
Tor seurs des efforts de cohésion:

$0 \leq x < \frac{L}{2}$



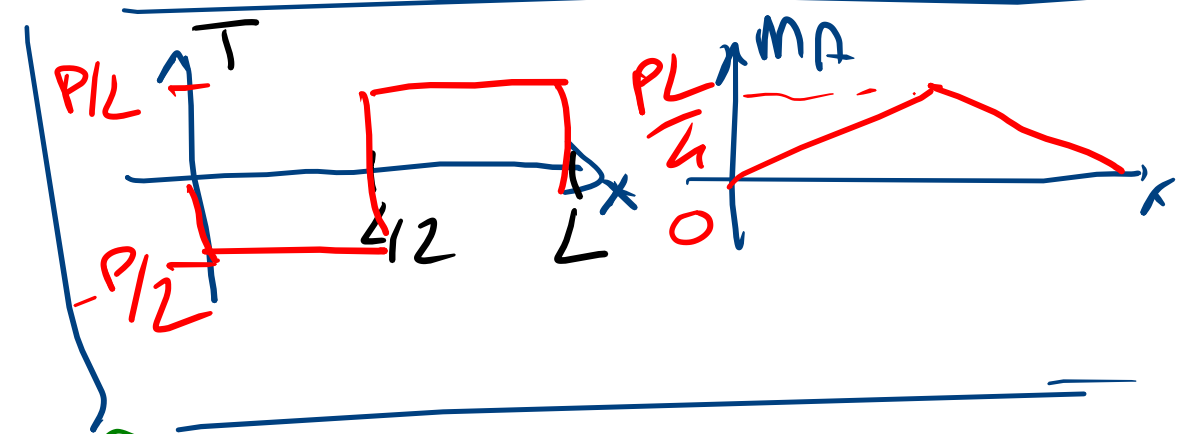
$\{ \mathcal{L}_{Ext \rightarrow \Sigma_-} \} = \left\{ \begin{array}{l} \vec{R}_{Ext \rightarrow \Sigma_-} = \frac{P}{2} \vec{y} \\ \vec{M}(G, \vec{R}_{Ext \rightarrow \Sigma_-}) = -\frac{P}{2} x \vec{z} \end{array} \right\}_G$

$\Rightarrow \{ \mathcal{L}_{coh} \}_G = \left\{ \begin{array}{l} -\frac{P}{2} \vec{y} \\ \frac{P}{2} x \vec{z} \end{array} \right\}_G$

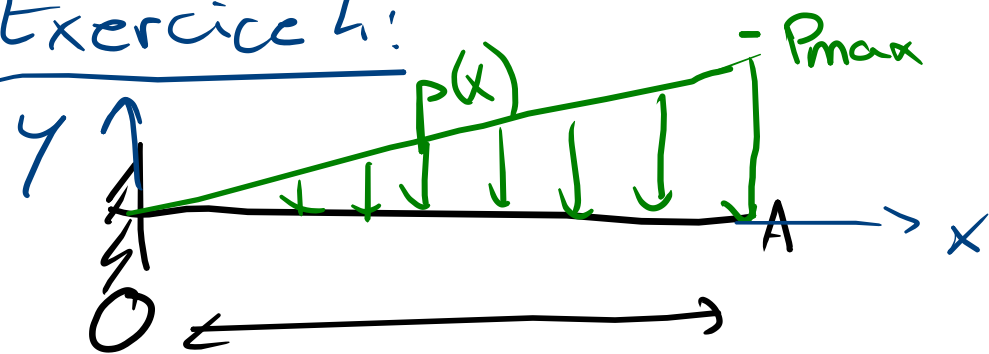


$\{ \mathcal{L}_{Ext \rightarrow \Sigma_-} \} = \left\{ \begin{array}{l} \vec{R}_{Ext \rightarrow \Sigma_-} = \frac{P}{2} \vec{y} - P \vec{y} = -\frac{P}{2} \vec{y} \\ \vec{M}(G, \vec{R}_{Ext \rightarrow \Sigma_-}) = -\frac{P}{2} L \vec{z} + P(x - \frac{L}{2}) \vec{z} \end{array} \right\}_G$

$\Rightarrow \{ \mathcal{L}_{coh} \}_G = \left\{ \begin{array}{l} \frac{P}{2} \vec{y} \\ -P(x - L) \vec{z} \end{array} \right\}_G$



Exercice 4:



Il faut définir la fⁿ $p: x \mapsto p(x)$

fⁿ affine: $x=0 \mapsto p(0)=0$

$x=L \mapsto p(L)=p_{\max}$

$$\Rightarrow \vec{p}(x) = -\frac{p_{\max}}{L} x \vec{y}$$

B.A.T.E. Liaison encastrement en O $\Rightarrow \left\{ \begin{array}{l} \vec{R}_0 = X_0 \vec{x} + Y_0 \vec{y} \\ \vec{T}(0, L_0) = N_0 \vec{z} \end{array} \right\}_O$

Charge répartie $\Rightarrow \left\{ \begin{array}{l} \vec{R}_p = \int_0^L \vec{p}(l) dl \\ \vec{T}(0, \vec{p}) = \int_0^L \vec{OT}_l \wedge \vec{p}(l) dl \end{array} \right\}_O$

$$\left\{ \begin{array}{l} \vec{R}_p = -\frac{p_{\max}}{2} L \vec{y} \\ \vec{T}(0, \vec{p}) = -\frac{p_{\max}}{3} L^2 \vec{z} \end{array} \right\}_O$$

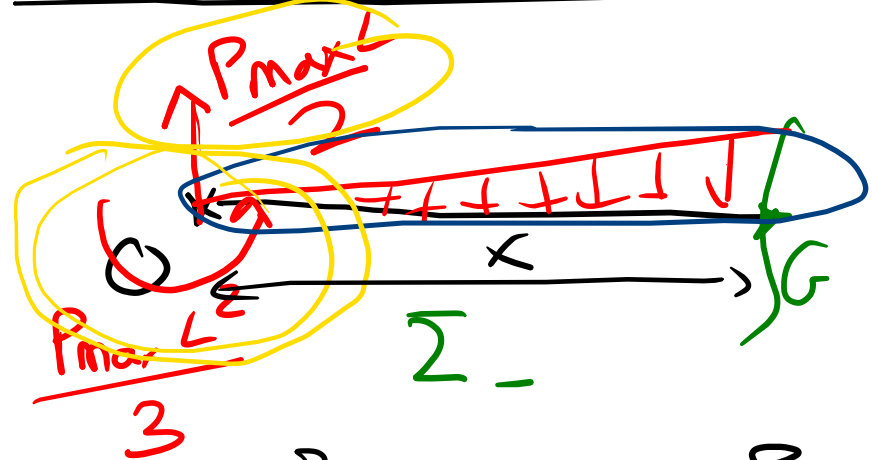
PFS:

$$X_0 = 0$$

$$N_0 = \frac{p_{\max}}{3} L^2$$

$$Y_0 = \frac{p_{\max}}{2} L$$

Torseur des efforts de cohésion: $0 \leq x \leq L$



$$\{ \mathcal{C}_{Ext \rightarrow \Sigma^-} \}_G = - \{ \mathcal{C}_{coh} \}_G$$

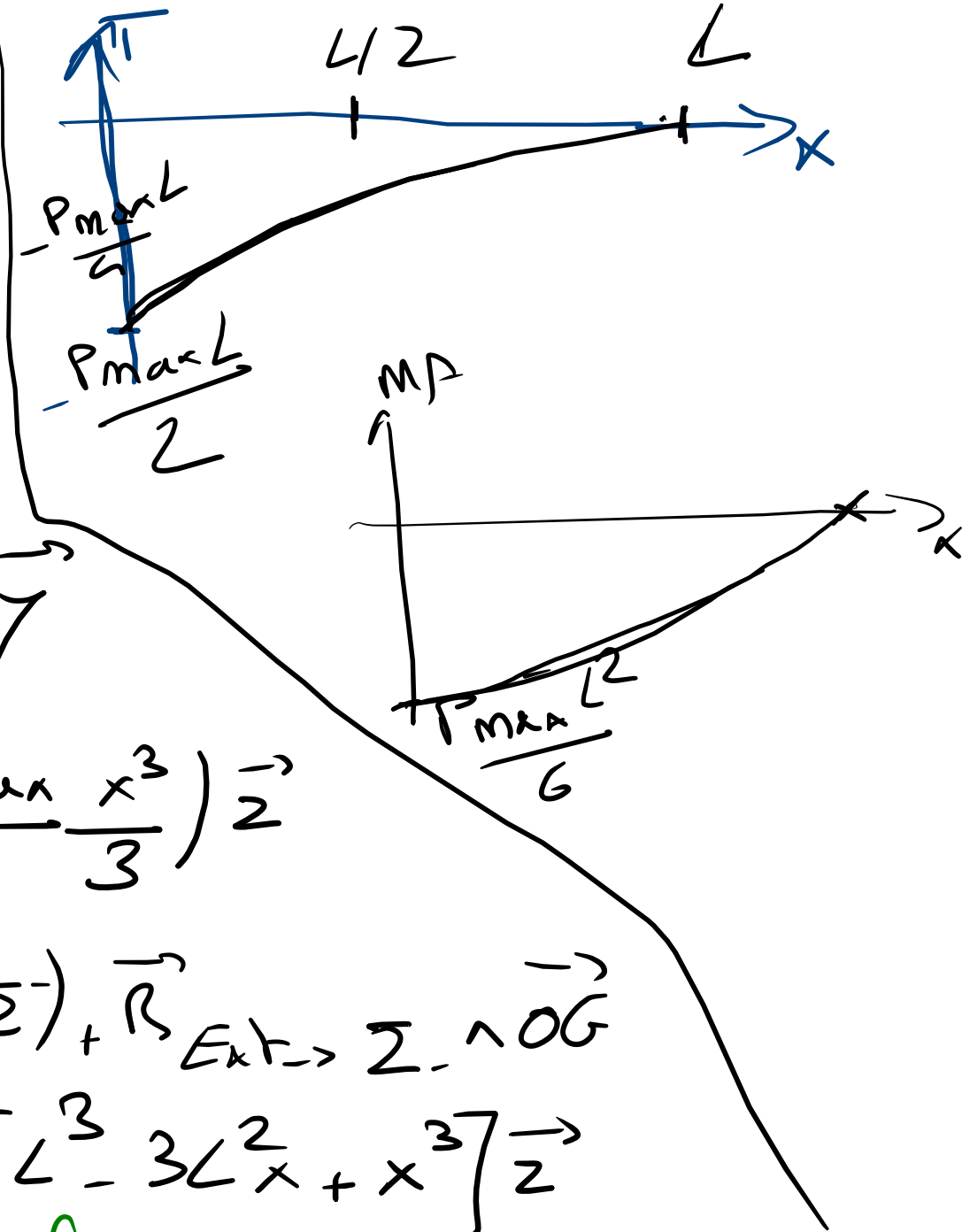
$$\vec{R}_{Ext \rightarrow \Sigma^-} = \frac{p_{max} L}{2} + \int_0^L \vec{p}(l) dl = \frac{p_{max}}{2L} (L^2 - x^2) \vec{y}$$

$$\vec{\mathcal{M}}(O, Ext \rightarrow \Sigma^-) = \frac{p_{max} L^2}{3} + \int_0^L O\vec{M}_A \vec{p}(l) dl = \left(\frac{p_{max}}{3} L^2 - \frac{p_{max}}{L} \frac{x^3}{3} \right) \vec{z}$$

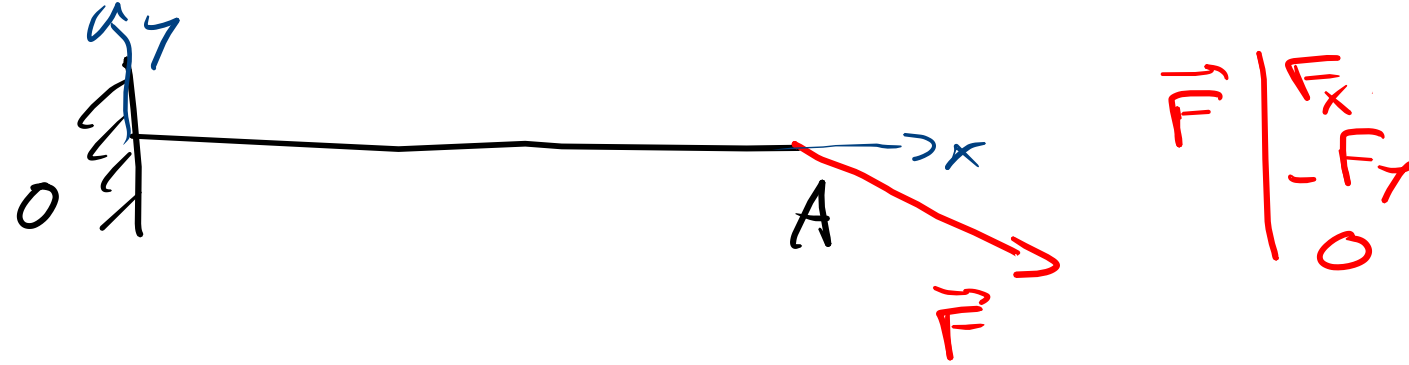
Transport de moment en G : $\vec{\mathcal{M}}(G, Ext \rightarrow \Sigma^-) = \vec{\mathcal{M}}(O, Ext \rightarrow \Sigma^-) + \vec{R}_{Ext \rightarrow \Sigma^-} \wedge \vec{OG}$

$$= \frac{p_{max}}{6L} [L^3 - 3L^2 x + x^3] \vec{z}$$

$$\{ \mathcal{C}_{coh} \}_G = \left\{ \begin{array}{l} \vec{R}_{coh} = \frac{p_{max}}{2L} (x^2 - L^2) \vec{y} \\ \vec{\mathcal{M}}(G, coh) = - \frac{p_{max}}{6L} [L^3 - 3L^2 x + x^3] \vec{z} \end{array} \right\}_G$$



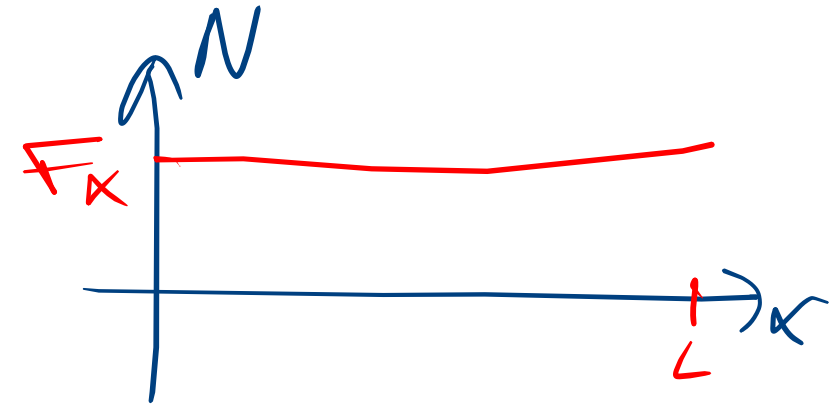
Exo1 modifié :



B.A.M.E :

Liaison encastrement en O : $\left\{ \begin{array}{l} \vec{R}_0 = x_0 \vec{x} + y_0 \vec{y} \\ \vec{M}(0, \vec{R}_0) = N_0 \vec{z} \end{array} \right\}_O$ $\left\{ \begin{array}{l} \vec{F} = F_x \vec{x} - F_y \vec{y} \\ \vec{M}(A, \vec{F}) = \vec{0} \end{array} \right\}_A$

PFS en O : $\boxed{X_0 = -F_x}$
 $Y_0 = F_y$ $N_0 = F_y L$



Corseur des efforts de cohésion

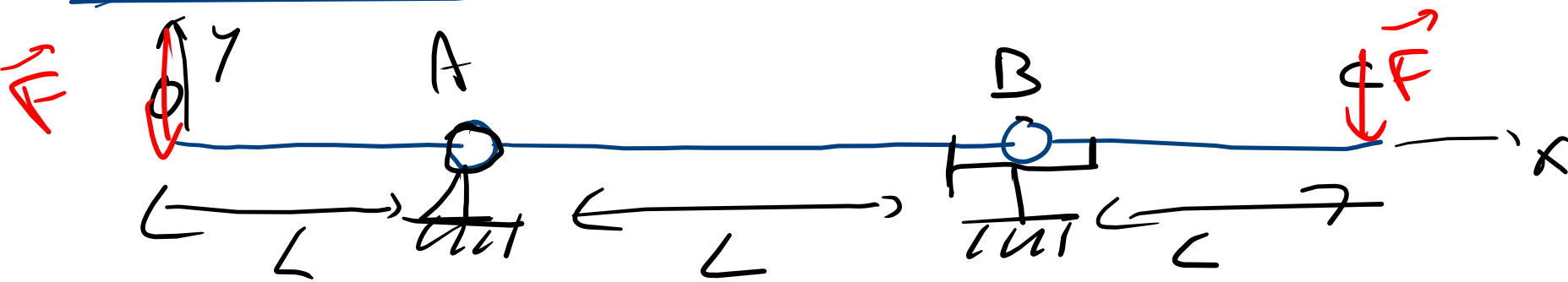
$$\left\{ \begin{array}{l} \vec{R}_{ext} = \vec{R}_- \\ \vec{M}_{ext} = \vec{M}_- \end{array} \right\} = \left\{ \begin{array}{l} -F_x \vec{x} + F_y \vec{y} \\ F_y L \vec{z} - F_y x \vec{z} \end{array} \right\}_G$$

$$\left\{ \begin{array}{l} \vec{R}_{coh} = F_x \vec{x} + F_y \vec{y} \\ \vec{M}(L, coh) = F_y (x - L) \vec{z} \end{array} \right\}_G$$

effort normal N à la section droite de poutre
 effort tranchant T
 moment fléchissant



Exercise 5:



$$\overbrace{B.A.\Pi.E} \quad \left\{ \sum \vec{\alpha}_A \right\} = \begin{cases} \vec{R}_A = x_A \vec{x} + Y_A \vec{y} \\ \vec{r}(A, \vec{\alpha}_A) \cdot \vec{z} = 0 \end{cases} / A$$

$$\left\{ \sum \vec{\alpha}_B \right\} = \begin{cases} \vec{R}_B = Y_B \vec{y} \\ \vec{r}(B, \vec{\alpha}_B) = \vec{0} \end{cases} / B \quad \left\{ \sum \vec{F} \right\} \quad \left\{ \sum \vec{M} \right\}$$

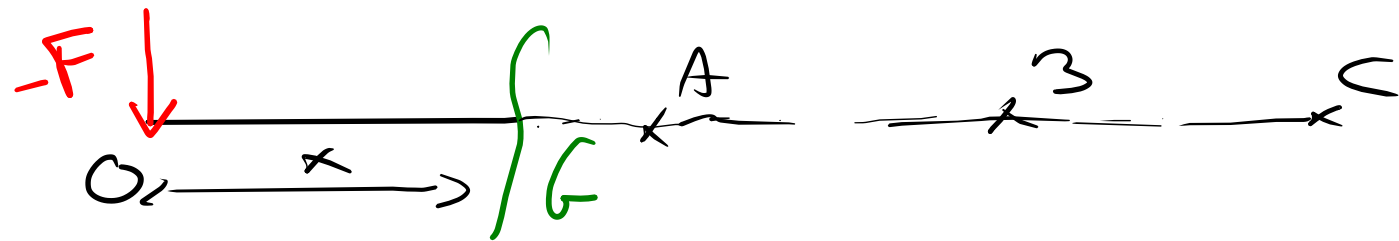
PFS: $x_A = 0 \quad Y_A = Y_B = F$

Torsion des efforts de cohésion:

3 cas de figures pour les efforts de cohésion

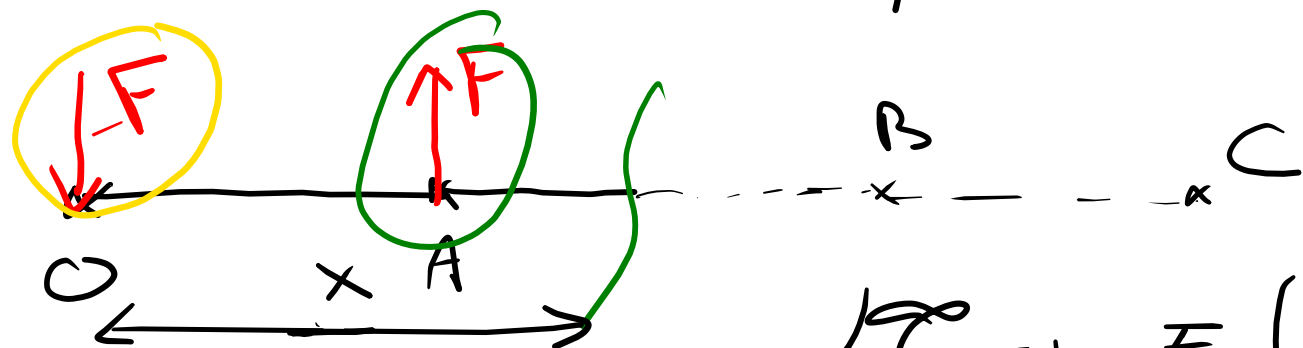
$$\underbrace{0 \leq x \leq L}_{\bar{E}_1} ; \underbrace{L \leq x \leq 2L}_{\bar{E}_2} ; \underbrace{2L \leq x \leq 3L}_{\bar{E}_3}$$

* $0 \leq x \leq L$:



$$\left\{ \mathcal{L}_{coh} \right\}_G = - \left\{ \mathcal{L}_{Ext \rightarrow \bar{E}_1} \right\}_G = - \left\{ \begin{array}{l} \vec{R}_{Ext \rightarrow \bar{E}_1} = -F\vec{y} \\ \vec{\eta}(G, Ext \rightarrow \bar{E}_1) = +F \times \vec{z} \end{array} \right\}_G \Rightarrow \left\{ \mathcal{L}_{coh} \right\}_G = \left\{ \begin{array}{l} F\vec{y} \\ -F \times \vec{z} \end{array} \right\}_G$$

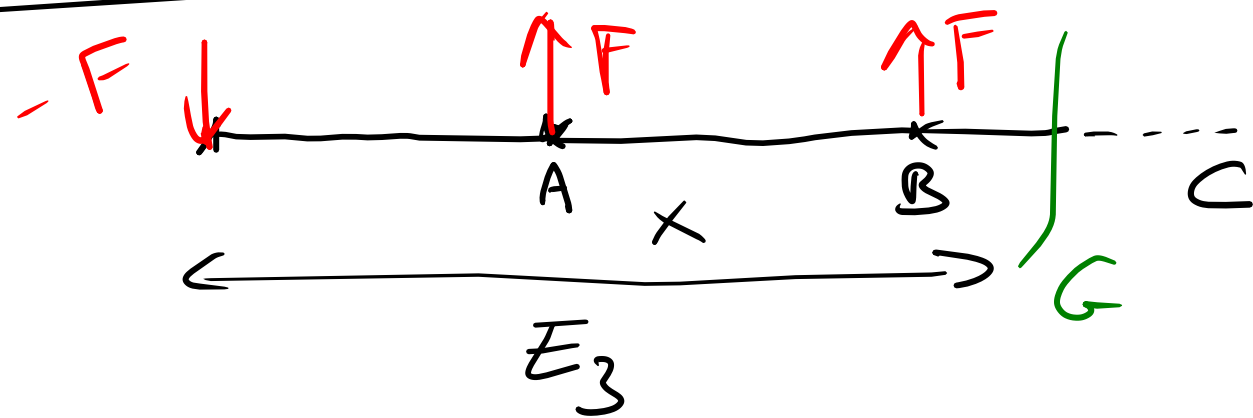
* $L \leq x \leq 2L$:



$$\left\{ \mathcal{L}_{Ext \rightarrow \bar{E}_2} \right\}_G = \left\{ \begin{array}{l} \vec{R}_{E \rightarrow \bar{E}_2} = -\underline{F\vec{y}} + \underline{F\vec{y}} = \vec{0} \\ \vec{\eta}(G, E \rightarrow \bar{E}_2) = \vec{\eta}(G, \vec{F}_O) + \vec{\eta}(G, \vec{F}_A) \\ = \vec{GO}_A F\vec{y} + \vec{GA}_A F\vec{y} \\ = (\vec{GO} + \vec{GA})_A F\vec{y} \\ = \vec{OA}_A F\vec{y} = FL\vec{z} \end{array} \right.$$

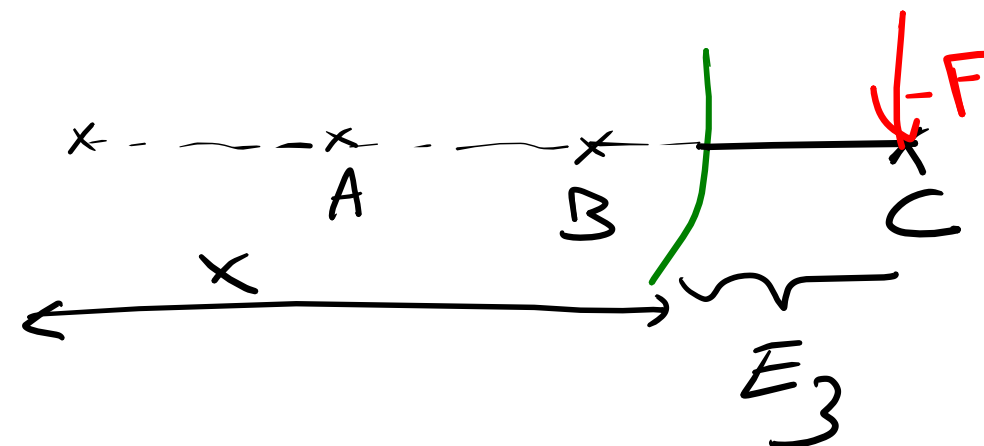
$$\left\{ \mathcal{L}_{coh} \right\}_G = - \left\{ \mathcal{L}_{Ext \rightarrow \bar{E}_2} \right\}_G = \left\{ \begin{array}{l} \vec{R}_{coh} = \vec{0} \\ \vec{\eta}(G, coh) = -FL\vec{z} \end{array} \right\}_G$$

$$\underline{2L \leq x \leq 3L:}$$



$$\{ \mathcal{L}_{coh} \}_G = - \{ \mathcal{L}_{Ext \rightarrow E_3} \}_G$$

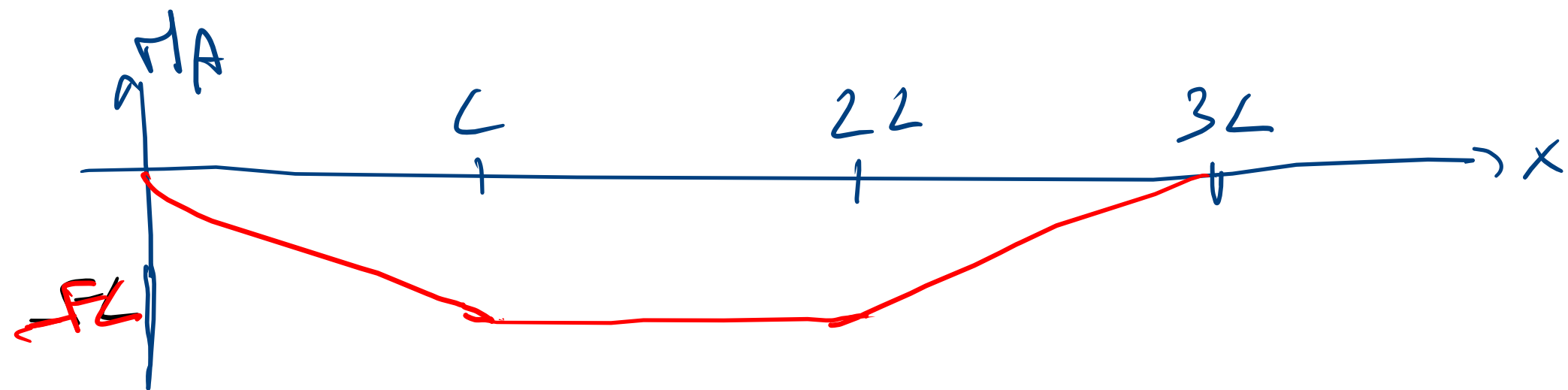
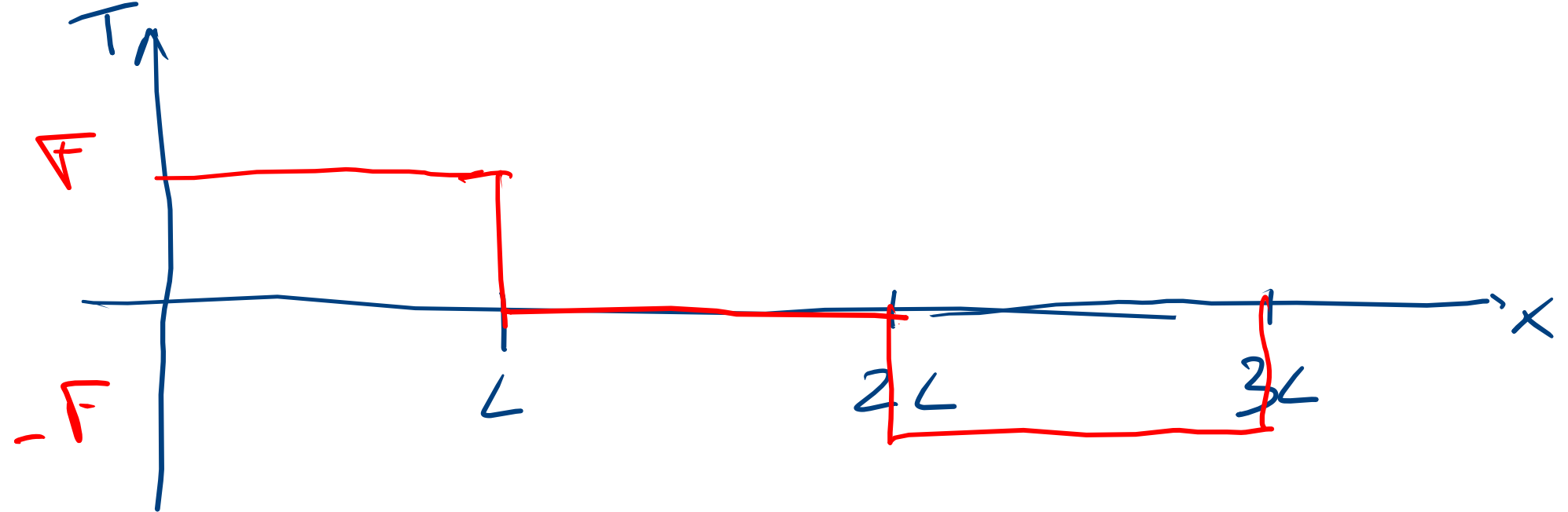
ou



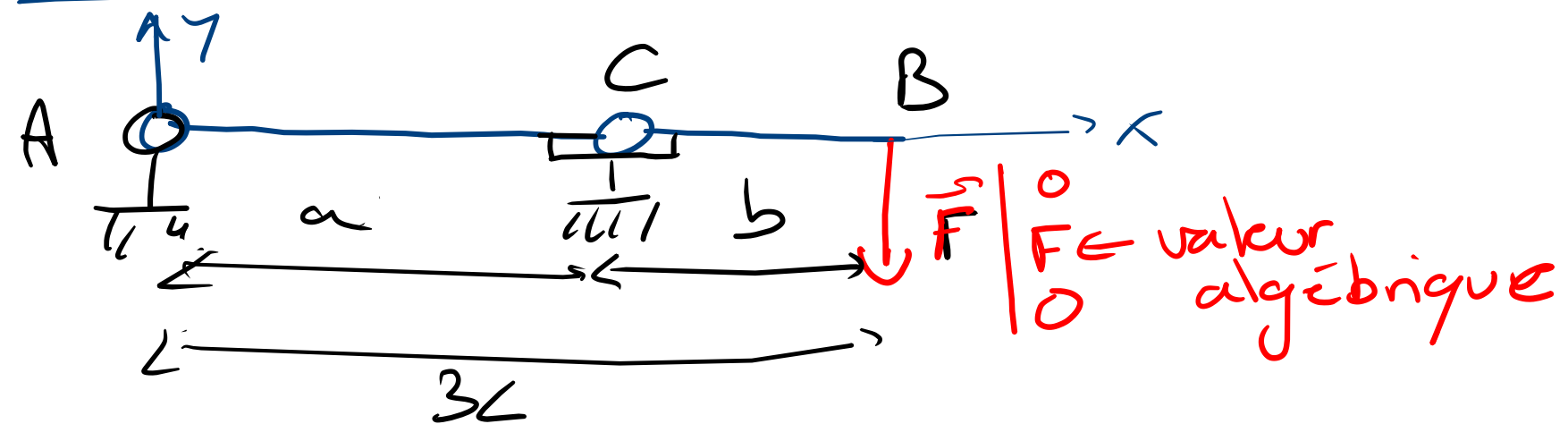
$$\{ \mathcal{L}_{coh} \}_G = \{ \mathcal{L}_{Ext \rightarrow E_3} \}$$

$$\{ \mathcal{L}_{Ext \rightarrow E_3} \}_G = \begin{cases} \vec{r}_{E \rightarrow E_3} = -F\vec{y} \\ \vec{\gamma}(G, \vec{r}_{E \rightarrow E_3}) = \vec{GC}_x - F\vec{y} \\ = -F(3L-x)\vec{z} \end{cases}$$

$$\{ \mathcal{L}_{coh} \} = \begin{cases} -F\vec{y} \\ F(x-3L)\vec{z} \end{cases}$$



Exercice 6.

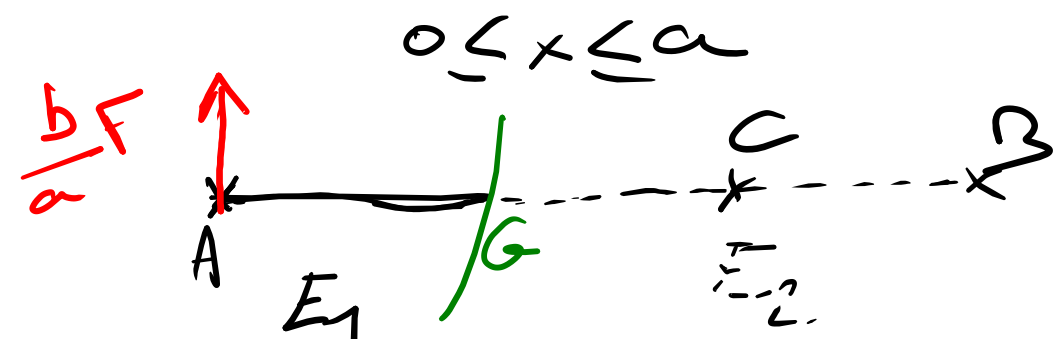


Réactions aux appuis en A et C

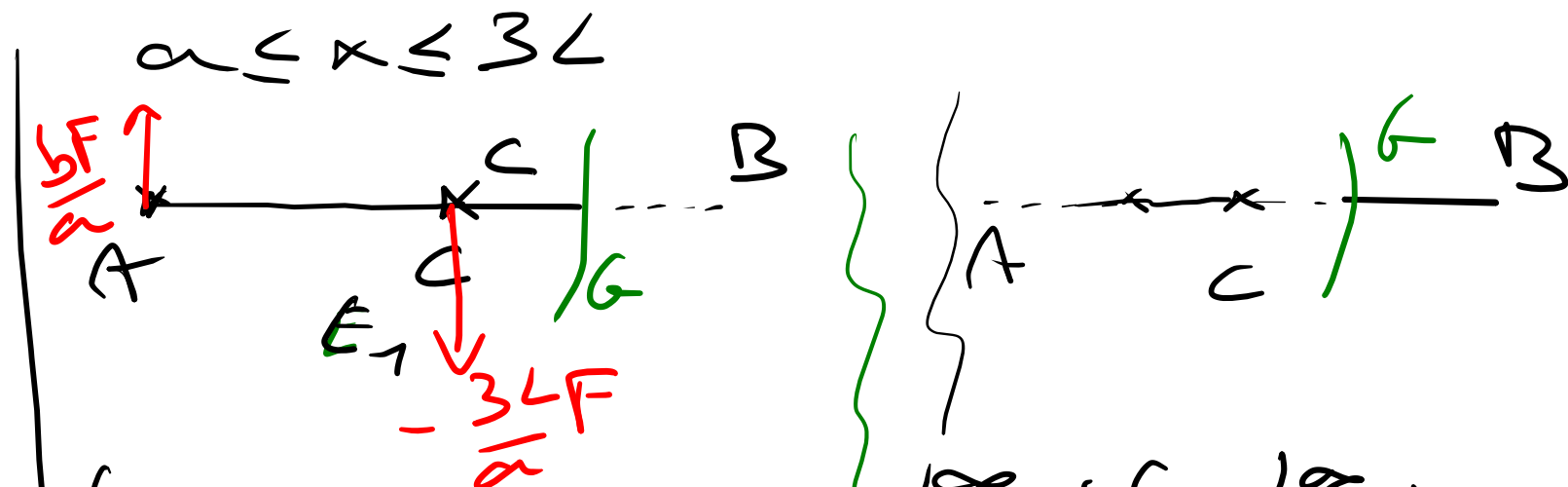
$$\text{PFS} \rightarrow X_A = 0$$

$$Y_A = \frac{b}{a} F \quad Y_C = -\frac{3L}{a} F$$

Torsion des efforts de cohésion:

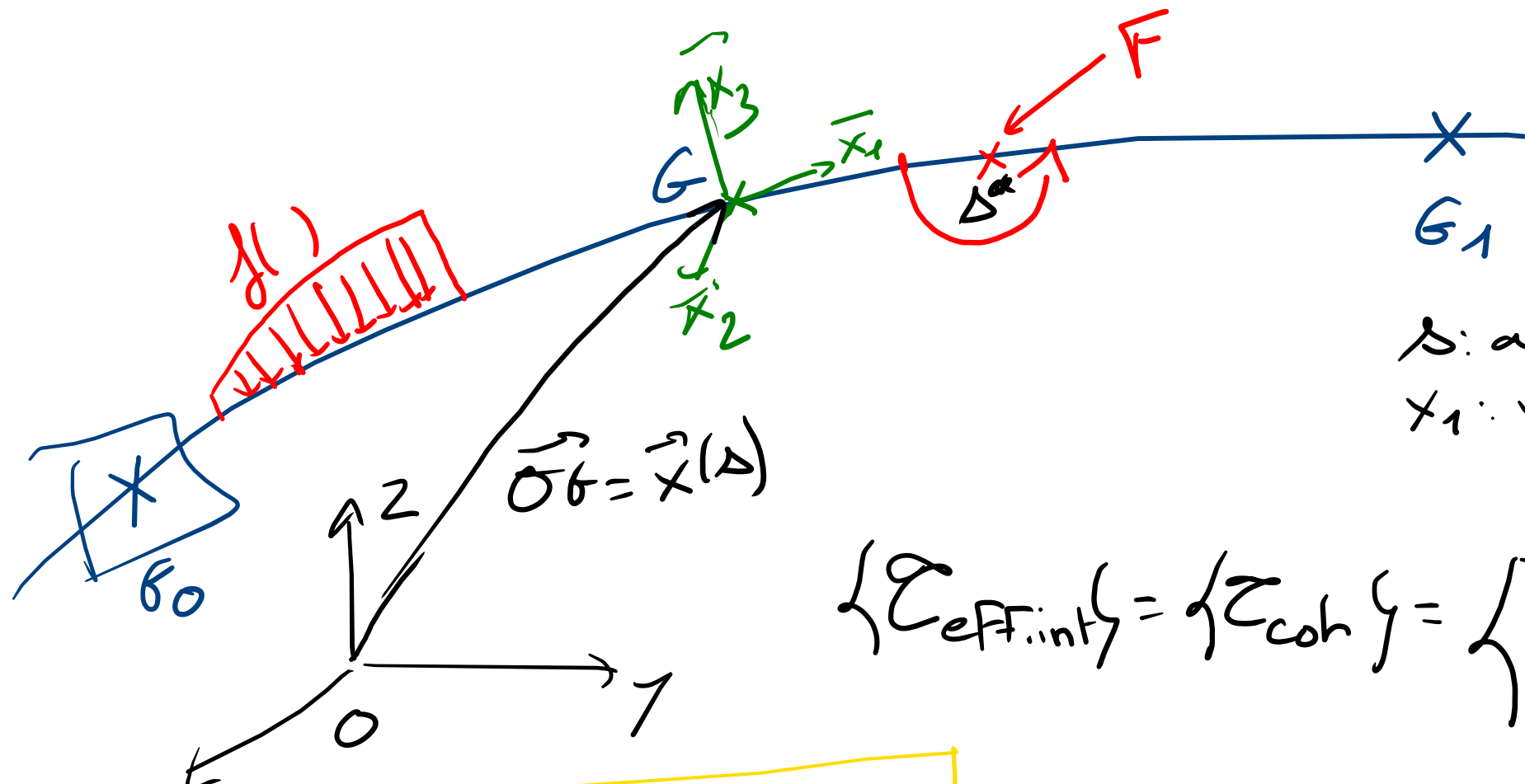


$$\{\mathcal{C}_{coh}\} = -\{\mathcal{C}_{Ext \rightarrow E_1}\} = \begin{pmatrix} 0 & 0 \\ -\frac{b}{a}F & 0 \\ 0 & \frac{b}{a}F_x \end{pmatrix}_G$$



$$\{\mathcal{C}_{coh}\} = \{\mathcal{C}_{Ext \rightarrow E_1}\}_G \quad \{\mathcal{C}_{coh}\} = \{\mathcal{C}_{Ext \rightarrow E_2}\}_L$$

$$\{\mathcal{C}_{coh}\}_G = \begin{pmatrix} 0 & 0 \\ F & 0 \\ 0 & F(BL-x) \end{pmatrix}_G$$



s : abscisse curviligne
 x_1 : vecteur unitaire tangent
 $\vec{x}_1 = \frac{d\vec{OG}}{ds}$

Efforts tranchants

$$\{\mathcal{C}_{eff.int}\} = \{\mathcal{C}_{coh}\} = \begin{cases} \vec{R}(s) = N(s)\vec{x}_1 + \bar{T}_2(s)\vec{x}_2 + \bar{T}_3(s)\vec{x}_3 \\ \vec{\Pi}(s) = \bar{\Pi}_1(s)\vec{x}_1 + \bar{\Pi}_2(s)\vec{x}_2 + \bar{\Pi}_3(s)\vec{x}_3 \end{cases}$$

$\hookrightarrow$ effort normal $\hookrightarrow \bar{\Pi}_1$ de torsion $\hookrightarrow \bar{\Pi}_2, \bar{\Pi}_3$ de flexion

$$\begin{cases} \frac{d\vec{R}}{ds}(s) + \vec{f}(s) = \vec{0} \\ \frac{d\vec{\Pi}}{ds}(s) + \vec{x}_1 \wedge \vec{R}(s) = \vec{0} \end{cases}$$

$$\forall s \in]s_0, s^*[\cup]s^*, s_1]$$

en $s^* \Rightarrow$ condition de saut: $[\vec{\Pi}(s)]_{s^*} + \vec{C}(s^*) = \vec{0}$
 $[\vec{R}(s)]_{s^*} + \vec{F}(s^*) = \vec{0}$

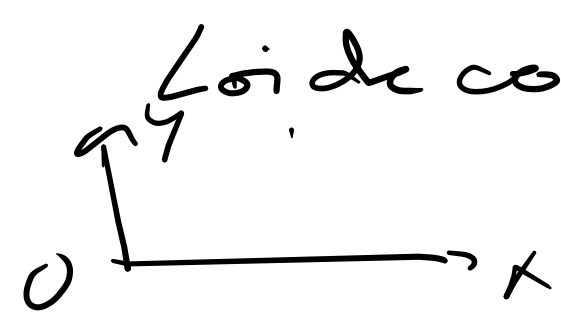
Condition aux limites en effort:

$$\begin{cases} \vec{R}(s_0) = -\vec{R}_{G_0} & \vec{R}(s_1) = \vec{R}_{G_1} \\ \vec{\Pi}(s_0) = -\vec{\Pi}_{G_0} & \vec{\Pi}(s_1) = \vec{\Pi}_{G_1} \end{cases}$$

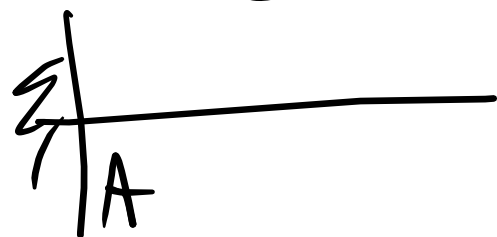
Calcul de la déformée:

(2D pour droite)

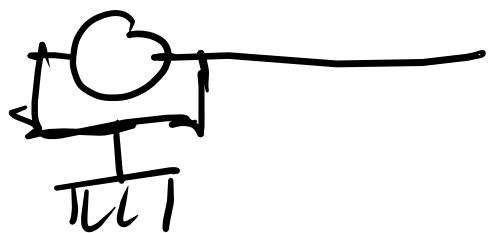
Loi de comportement
vd' suivant $\bar{y}$


$$\Rightarrow EI v''(x) = M_F(x)$$

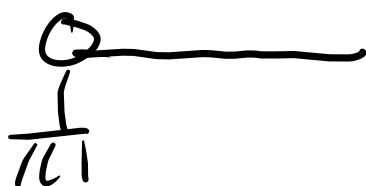
C. L endit



$$\begin{aligned} u_A &= 0 \\ v_A &= 0 \\ v'_A &= 0 \end{aligned}$$



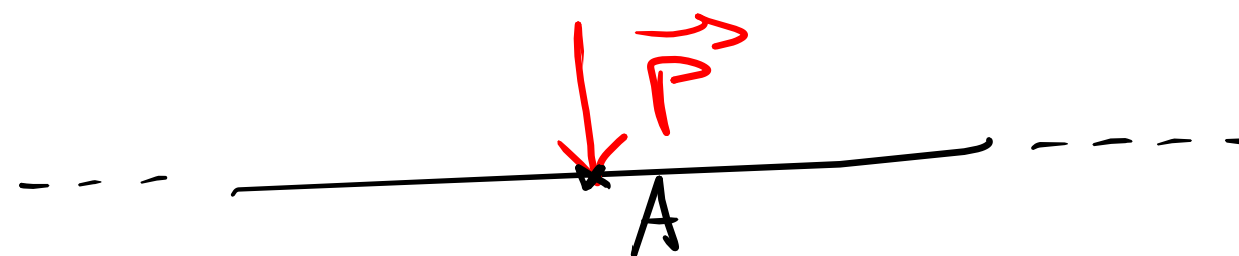
$$v_A = 0$$



$$u_A = 0$$

$$v_A = 0$$

conditions aux limites
en déplacement
et condition de raccord
condition de raccord



$$v_{A-} = v_{A+}$$

$$v'_{A-} = v'_{A+}$$