Differential calculus

$$\ell = L(u, v)$$

$$\ell(x) = L(u(x), v(x))$$

$$\frac{d\ell}{dx} = \frac{dL}{du}\frac{du}{dx} + \frac{dL}{dv}\frac{dv}{dx}$$

The differential is linear, variations add

Example $\ell(x) = \dfrac{u(x)}{v(x)}$

$$L(u, v) = \frac{u}{v}$$

$$\frac{d\ell}{dx} = \frac{dL}{du}u'(x) + \frac{dL}{dv}v'(x)$$

$$\frac{dL}{dx} = \frac{u'(x)}{v(x)} - \frac{u(x)}{v^2(x)}v'(x)$$

$$\frac{d\ell}{dx} = \frac{(u'(x)v(x) - v'(x)u(x))}{v^2(x)}$$