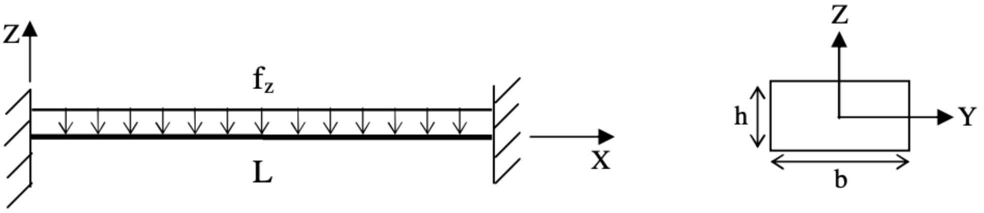
Feuille de TD 3-EF :

I] Exercise 1.

An engineer wants to use finite element analysis to study the behaviour of a thin, rectangular-cross-section steel beam of length L. The structure is fixed at both ends (see Figure 1). The structure's own weight is not taken into account. The total load applied to the beam is F, which is a uniformly distributed load. *Note that Bernouilli conditions are considered* ($\theta(x) = \frac{dv(x)}{dt}$)



A] Analytical resolution.

- Establish the equilibrium equation, the behaviour law, the boundary conditions.
- Solve the problem analytically to obtain the displacement **v(x)** at any point.

B] Approximate resolution by the EF method.

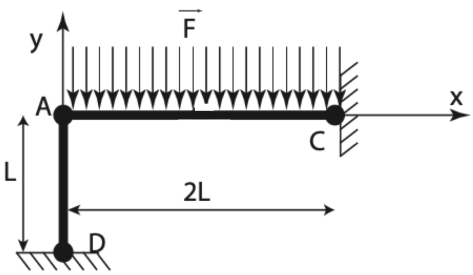
- Write the variational formulation of the problem.
- Recall the nodal unknowns of the beam element.
- Recall the variable transformation used to switch from the x-coordinate system to the r-coordinate system.
- Recall the expressions of the 4 interpolations functions ($\mathrm{N_1(r)}$, $\mathrm{N_2(r)}$, $\mathrm{N_3(r)}, \mathrm{N_4(r)}$) in the reference coordinate system.
- Show that the stiffness matrix for one element can be written in the following form (to do so, express the $[B_{\text{ref}}]$ vector and use the change of variables from the r-reference frame to the x-frame). Note that during the tutorial session we will compute only K_{11}. You will compute the remaining components of the matrix at home.

$$[K]_e = \left(\frac{EI_z}{L_e^3}\right) \begin{bmatrix} 12 & 6L_e & -12 & 6L_e \\ & 4L_e^2 & -6L_e & 2L_e^2 \\ & & 12 & -6L_e \\ sym & & & 4L_e^2 \end{bmatrix}$$

- Construct $[\mathrm{F_{ref}}]$ and deduce $[\mathrm{F_e}]$.
- Assemble the system, apply the boundary conditions, solve the system $[\mathrm{K}][\mathrm{Q}] = [\mathrm{F}]$

2] Exercise 2 – for practice if no time to do in classes: Study of a Frame (“portique“)

The objective of Exercise 2 is to study a frame (“Portique“ in french) composed solely of **full (i.e. including axial and bending effects)** beam elements. The structure consists of a horizontal beam of length $2L$, subjected to a distributed load $\vec{F} = -f\,\vec{Y}$ (with $f = 10$ N/mm^3), and a vertical beam of length $L = 0.5$ m. To simplify the expressions, you will assume that ES= 1Kg.L.s^{-2} and EI= Kg.L3.s^{-2} . We will consider **only two elements**: a horizontal one of length $2L$, labeled 1, and a vertical one of length L, labeled 2. The two elements are rigidly connected at point A.



1] Give the number of degrees of freedom for each node and total for the problem. Write the boundary conditions of the problem.

2] Recall the elemental stiffness matrix for (a) an element subjected only to axial tension/compression, and (b) an element subjected only to bending. From this, deduce the elemental stiffness matrix $[K]$ of a full beam element (axial and bending) in the local coordinate system of the beam.

3] Give the relationships between the degrees of freedom of elements (1) and (2) at point A. (Note: consider first the point A belonging to bar 1, then the point A belonging to bar 2, to write these relationships.)

4] Define the global vector of unknowns [Q_{global}]. Assemble and write the global stiffness matrix $[K]$ of the problem.

5] Write the vector $[F]$, the equivalent nodal force vector, of the problem.

6] Taking the boundary conditions into account, write and solve the system.