

Practical Gibbs Sampler

We consider a random vector $X = (X_1, X_2)$ taking values in $\{0, \dots, n\} \times [0, 1]$. The density of X on its support is proportional to

$$\frac{n!}{(n - x_1)!x_1!} x_2^{x_1 + \alpha - 1} (1 - x_2)^{n - x_1 + \beta - 1}.$$

1 For $n = 10$, $\alpha = 1$, and $\beta = 2$, implement a Gibbs sampler to generate samples approximately distributed according to the law of X . Use graphical diagnostics to verify the convergence of the resulting Markov chain to its stationary distribution.

2 Estimate $\mathbb{E}(X_1 X_2)$.